ECE 431
Digital Signal Processing
Homework 2

Due Friday, September 22, at beginning of class

Note: Feel free to work with other students on the homework, but you must hand in your own solutions and computer programs (identical answers are not allowed).

1. Brush-up on the FT and Convolution

- **a.** Show that the DTFT of $x[n] * y[n]$ is given by $X(\omega)Y(\omega)$, where $X(\omega)$ and $Y(\omega)$ are the DTFTs of $x[n]$ and $y[n]$, respectively.
- **b.** Suppose that $x(t)$ is a real-valued CT signal. What special property does its CTFT possess?
- **c.** Show that $w(t) * (x(t) + y(t)) = w(t) * x(t) + w(t) * y(t)$.
- **d.** Suppose that two DT LTI systems with impulse responses $g[n]$ and $h[n]$ are put in series. Show that the input-output behavior is independent of the order of the systems. That is, show that the input-output characteristic of $g[n]$ followed by $h[n]$ is identical to that of $h[n]$ followed by $g[n]$.

2. Let $x(t)$ be a CT signal and sample it at a rate of T samples/second. Recall that the CT sampled signal $x_s(t) = \sum_n x(nT)\delta(t - nT) = \sum_n x[n]\delta(t - nT)$. How is the DTFT of $x[n]$

$$X(\omega) = \sum_n x[n]e^{-j\omega n}$$

related to the CTFT of $x_c(t)$? Hint: Consider the expression for the CTFT of $x_s(t)$ and relate $X(\omega)$ to $X_s(\Omega)$ using the identification $\omega \equiv \Omega T$.

3. EKG (electrocardiogram) signals are approximately bandlimited to ± 20 Hz. EKG signals are often measured in the presence of strong 60Hz interference, so the recorded signal consists of the EKG signal of interest plus a 60Hz noise signal.

- **a.** Is it possible to eliminate the 60 Hz noise without any DT filtering. That is, can the noise be removed using only A/D and D/A converters, possibly with some minor modifications? If so, explain a method for doing this.
- **b.** Is it necessary to sample a rate above 120 Hz ? If not, explain how the noise can be removed even if it is undersampled, and state the minimum sampling rate you think is reasonable.

4. Consider a more realistic model for an A/D converter in which the *ideal impulse* sampling train is replaced by a train of *pulses* of the basic form

$$p(t) \;=\; \begin{cases} 0 & , \quad t < 0 \\ 1/\Delta & , \quad 0 \le t \le \Delta \\ 0 & , \quad t > \Delta \end{cases}$$

where $0 < \Delta < T$, where T is the sampling period. That is, instead of $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ we use $s_p(t) = \sum_{n=-\infty}^{\infty} p(t - nT)$. Mathematically characterize the impact of using pulses instead of ideal impulses in the frequency domain.