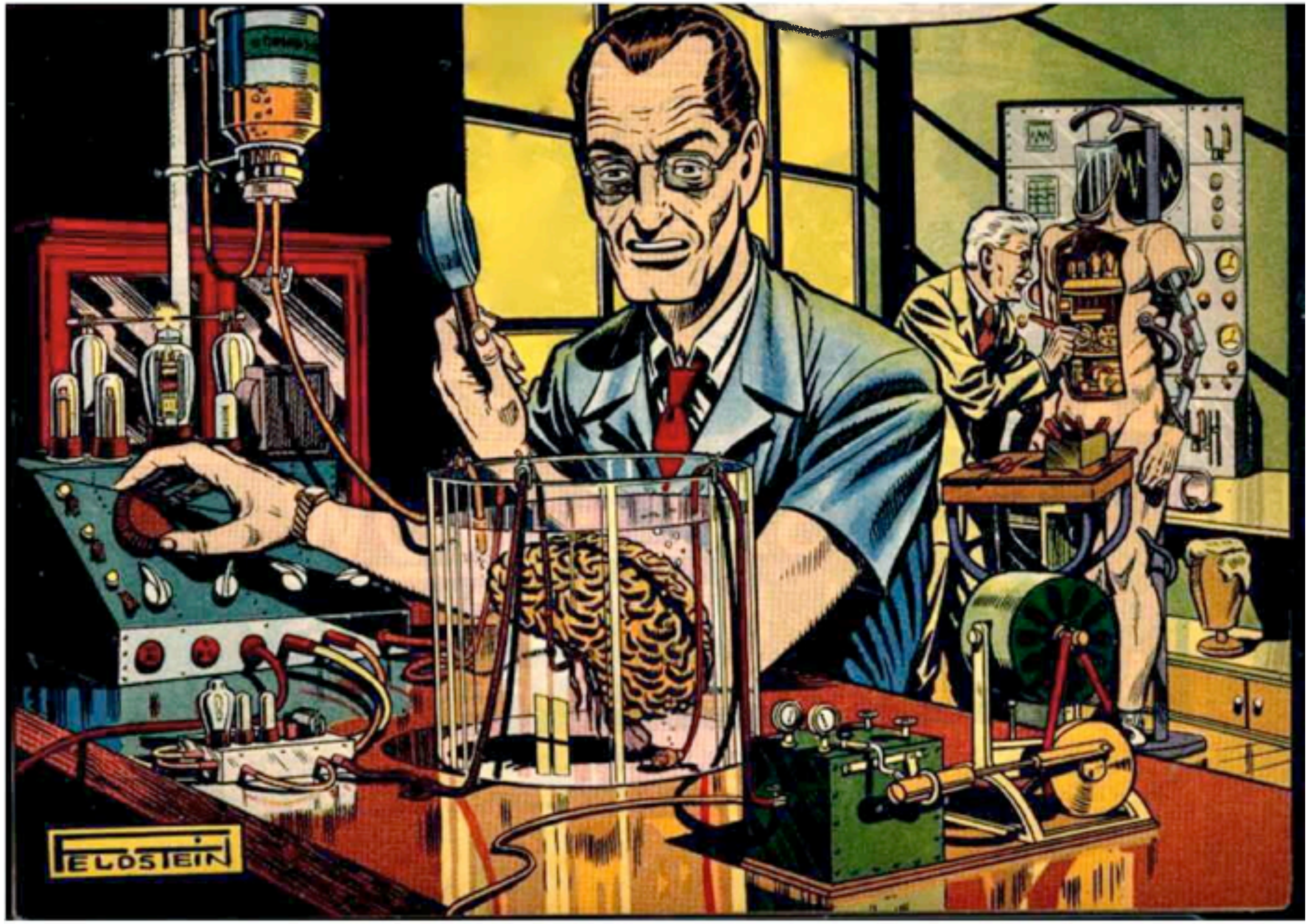
Active Sensing and Learning



ICASSP 2011, May 23, Prague

www.ece.wisc.edu/~nowak/ASL.html

Jarvis Haupt
www.ece.umn.edu/~jdhaupt

Rob Nowak
www.ece.wisc.edu/~nowak

Adaptive Information

Goal: Estimate an unknown object $x \in \mathcal{X}$ from scalar samples

Information: samples of the form $y_1(x), \ldots, y_n(x)$,
the values of certain functionals of x

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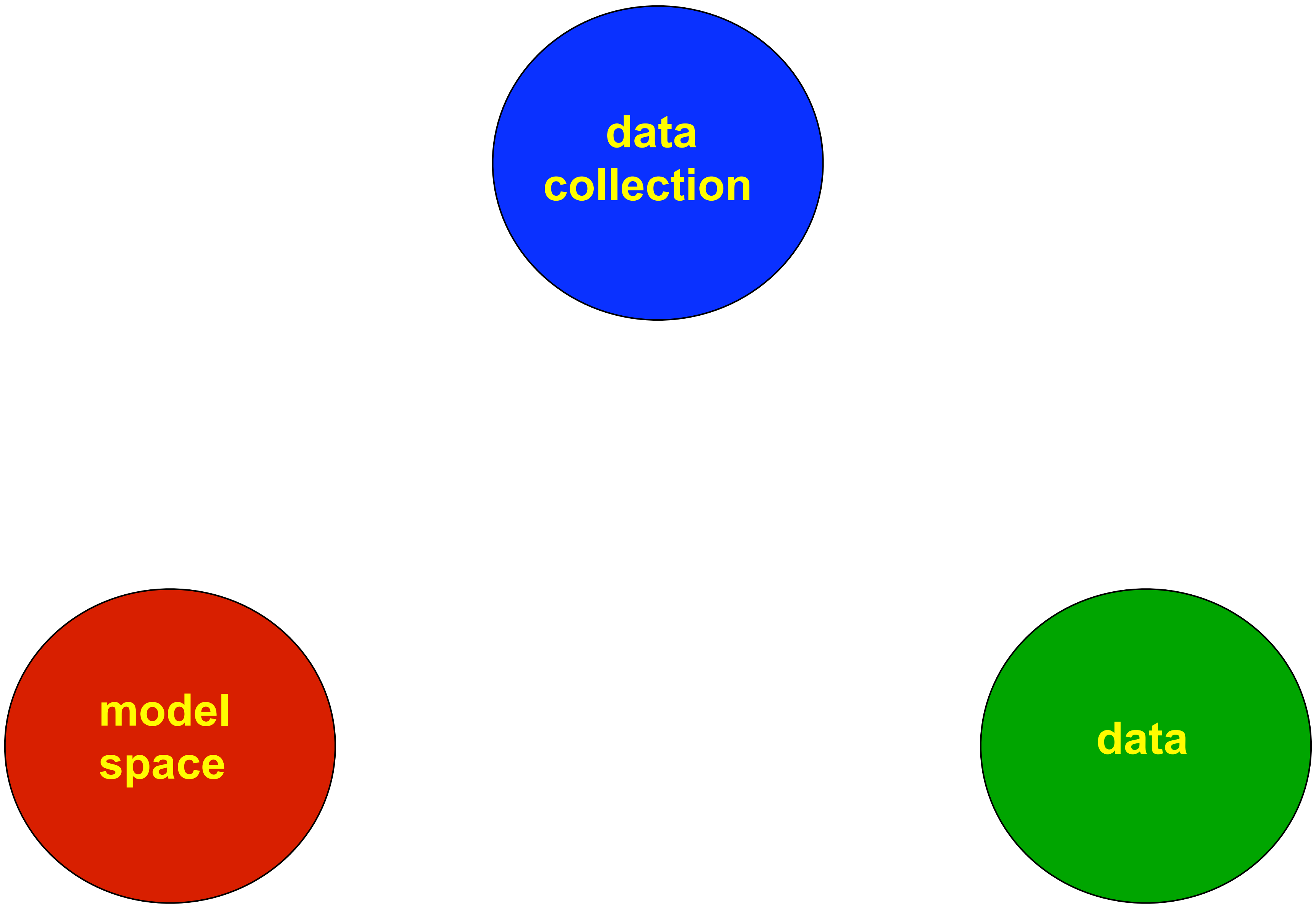
Does adaptivity help?

Feedback from Data Analysis to Data Collection

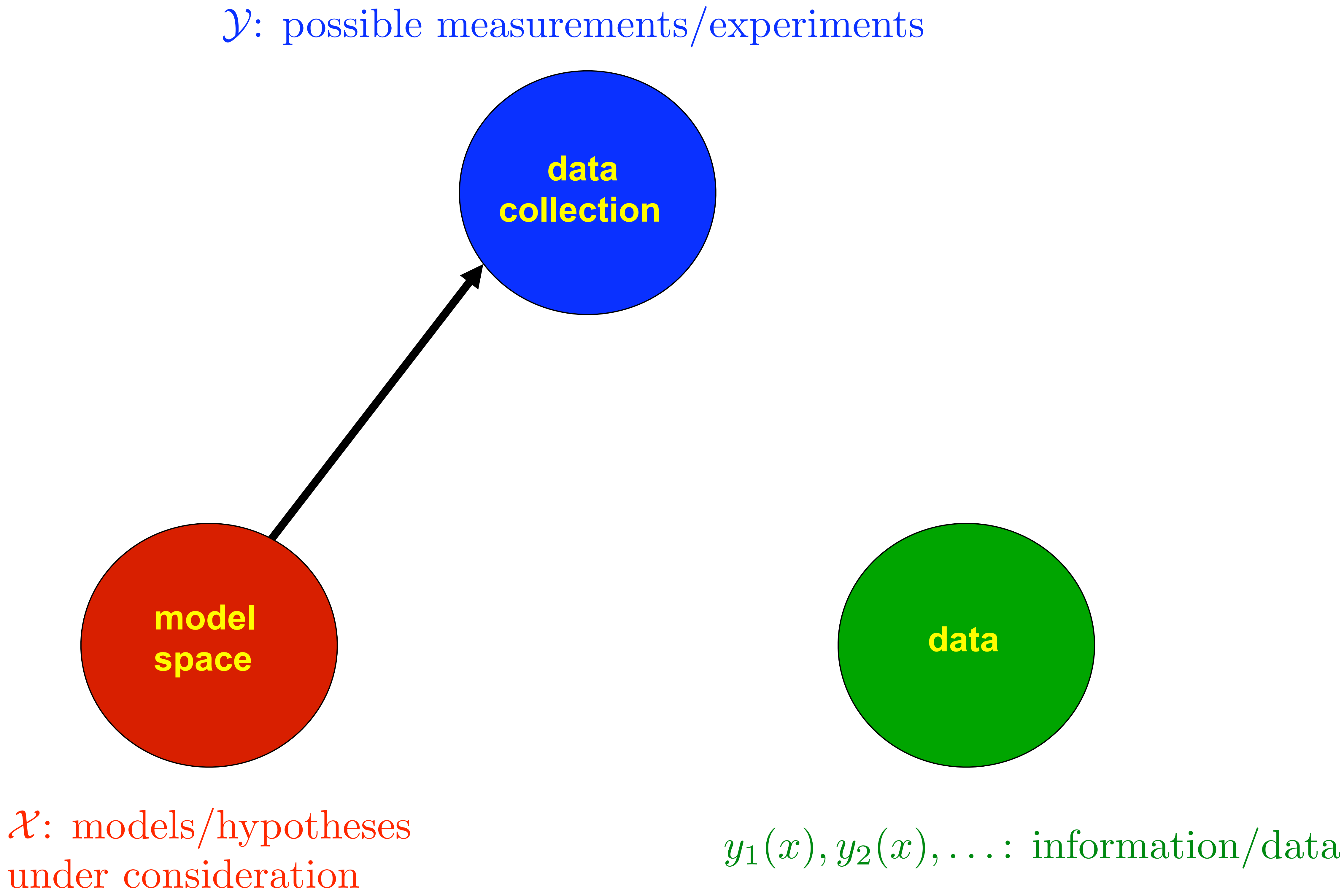
$\mathcal{Y}$: possible measurements/experiments



$\mathcal{X}$: models/hypotheses
under consideration

$y_1(x), y_2(x), \ldots$: information/data

Feedback from Data Analysis to Data Collection



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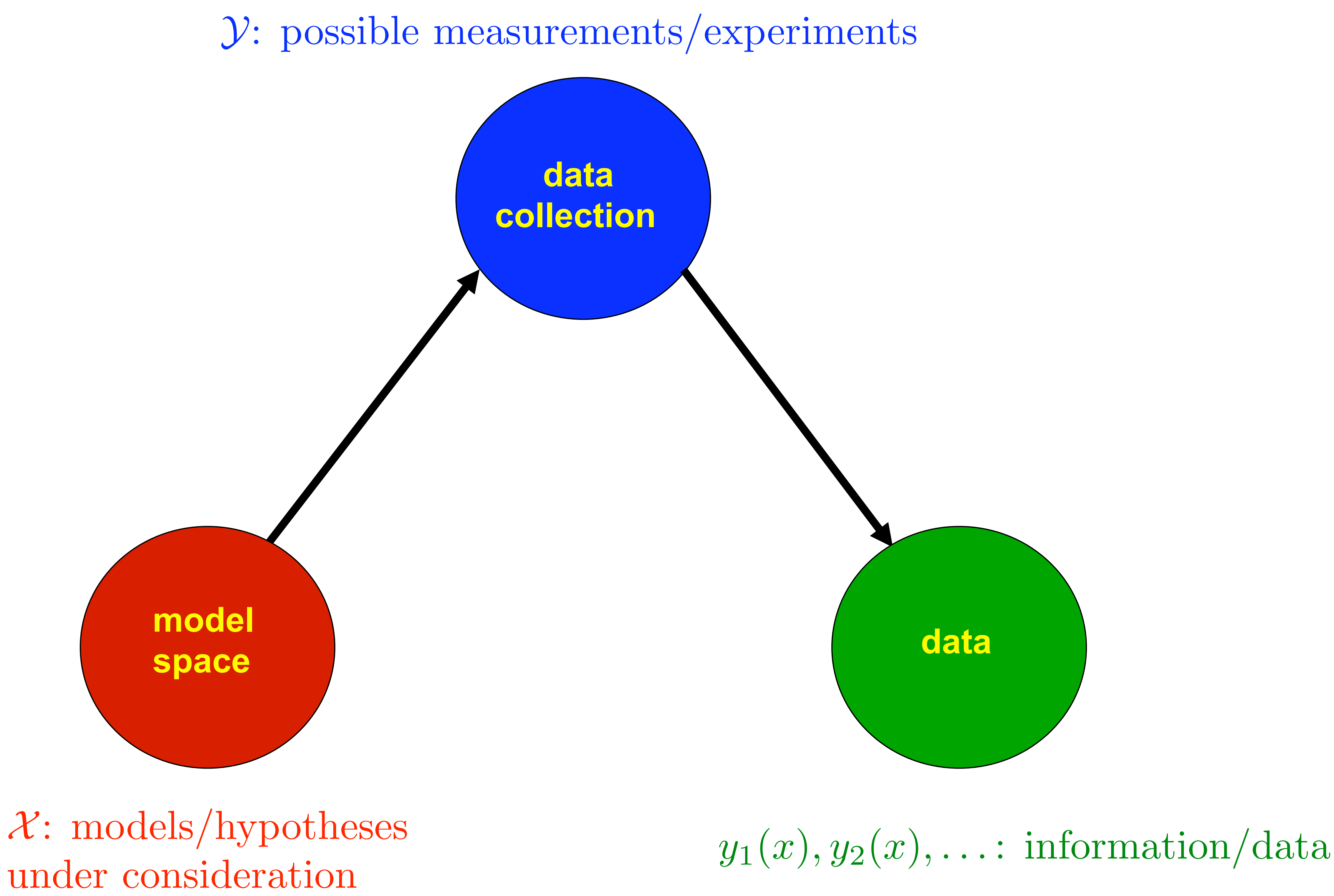

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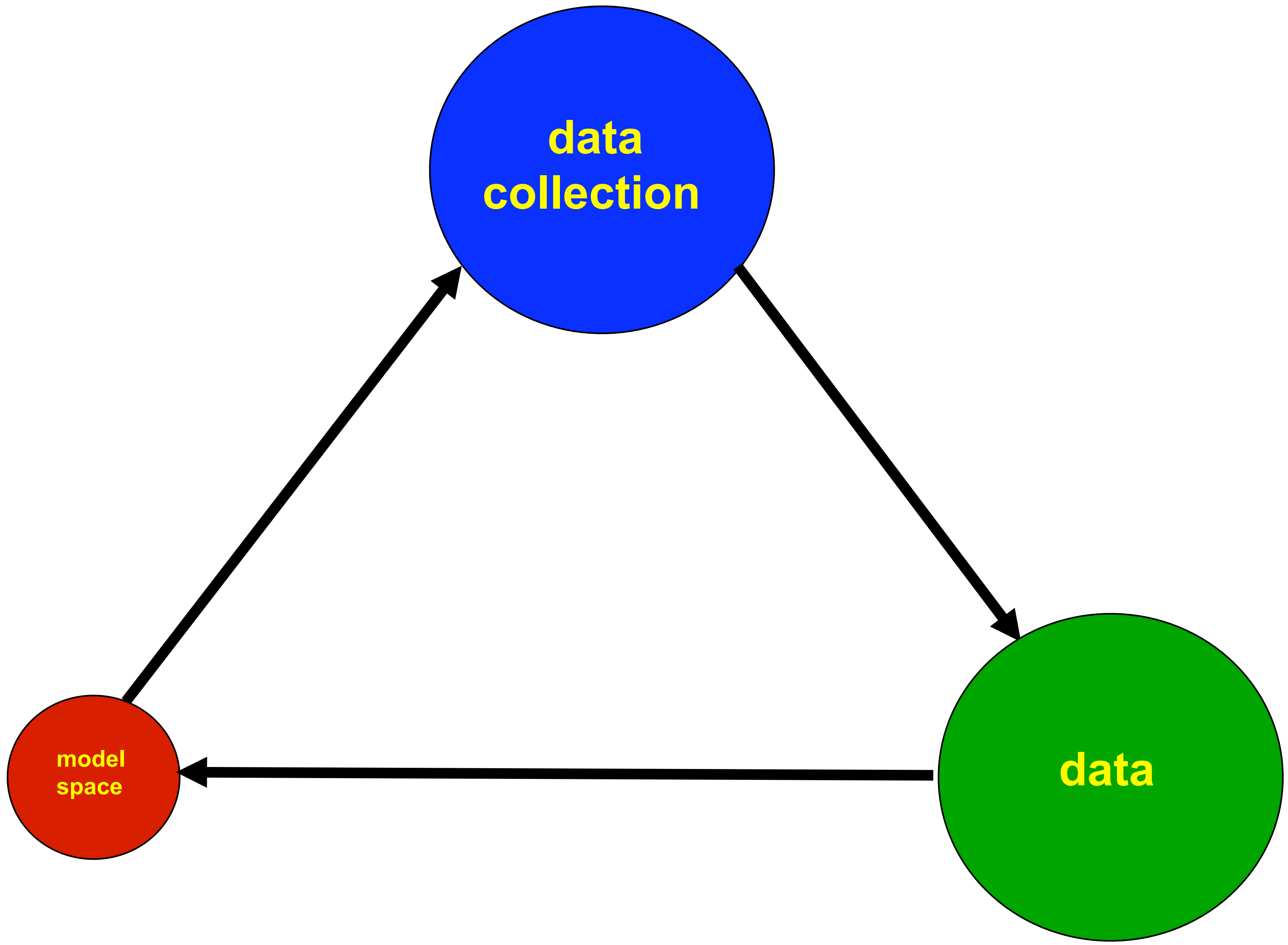

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Outline of Tutorial

Part 1: Introduction (Rob), 9:00-9:30

Part 2: Active Sensing (Jarvis), 9:30-10:30

Break, 10:30-10:45

Part 3: Active Learning (Rob), 10:45-11:45

Part 4: Conclusions and Future Directions, 11:45-12

Outline of Part 1:

Sequential Experimental Design

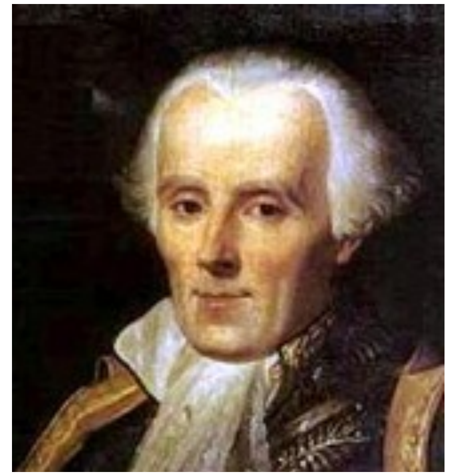
Adaptive Sensing for Sparse Recovery

Sensing and Inference in Large Networked Systems

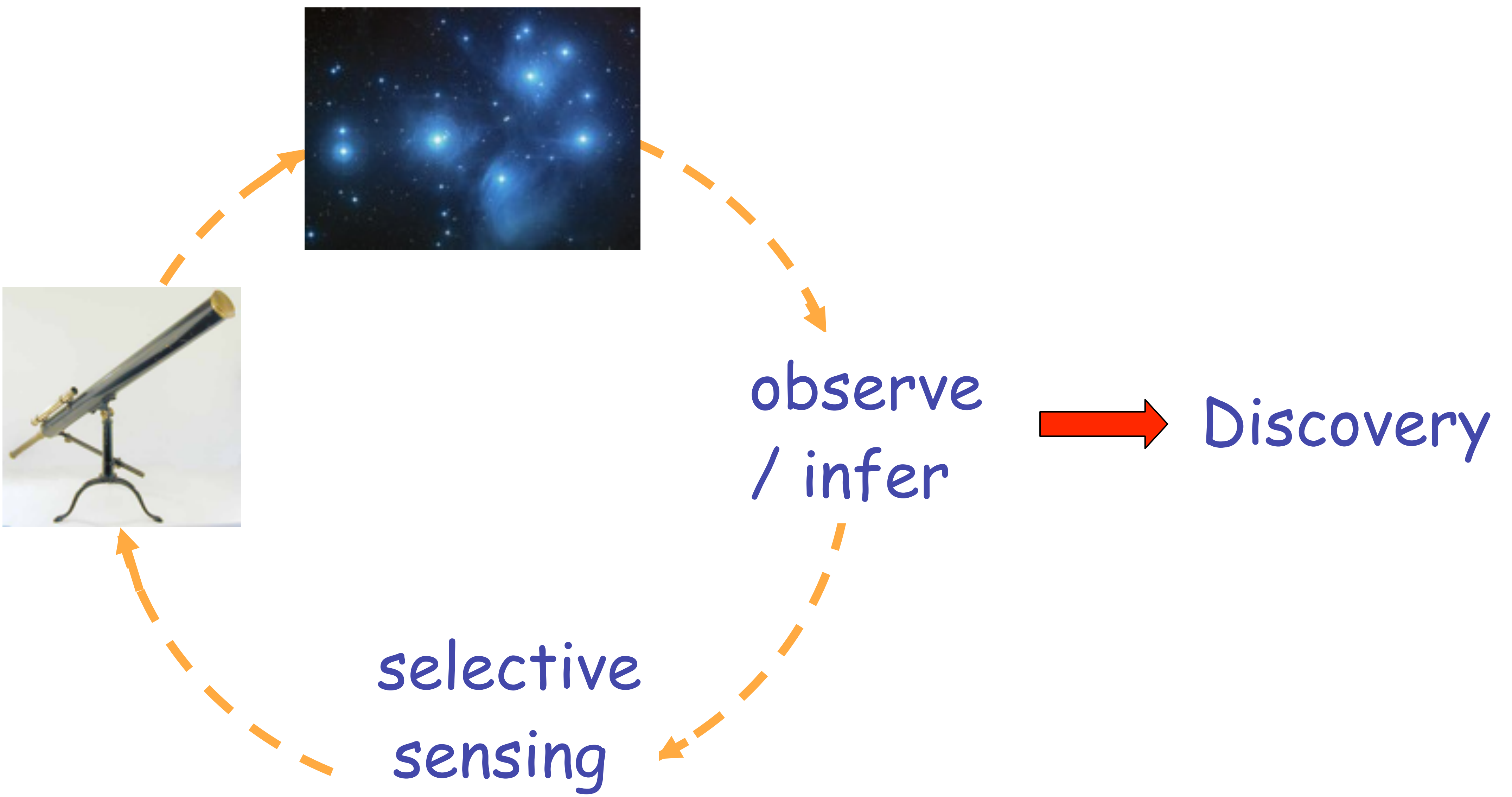
Active Learning in Machines and Humans

Mathematics of Active Sensing and Learning

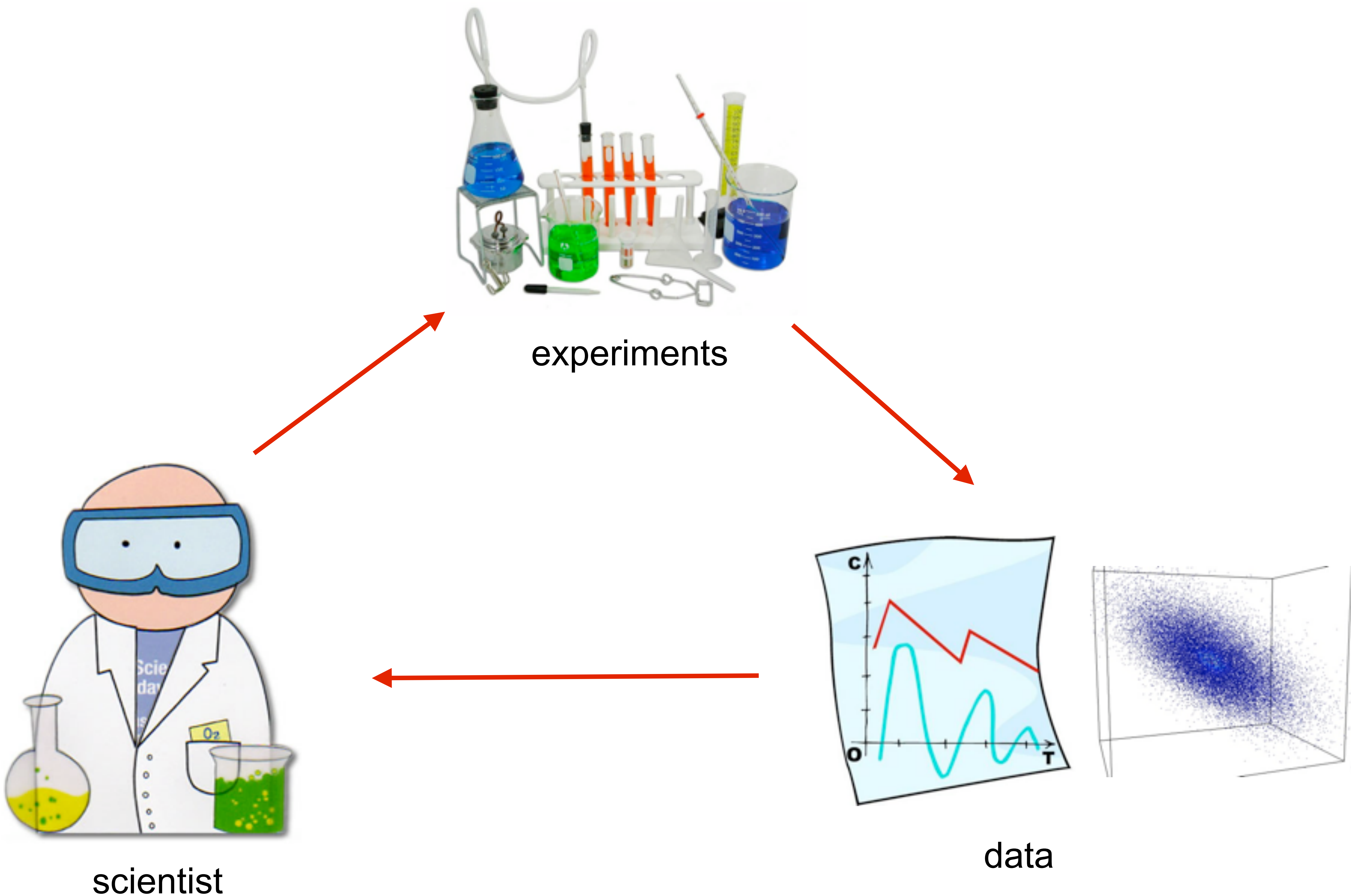
Sequential Experimental Design



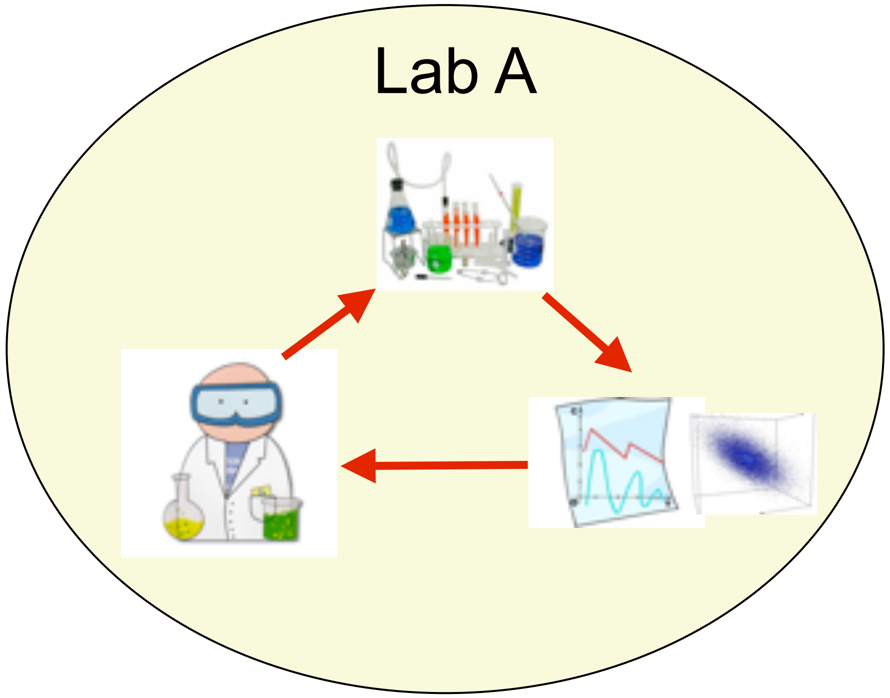
Decided to make new astronomical measurements when “the discrepancy between prediction and observation [was] large enough to give a high probability that there is something new to be found.” Jaynes (1986)



The Scientific Process in a Laboratory



The Scientific Process at Large



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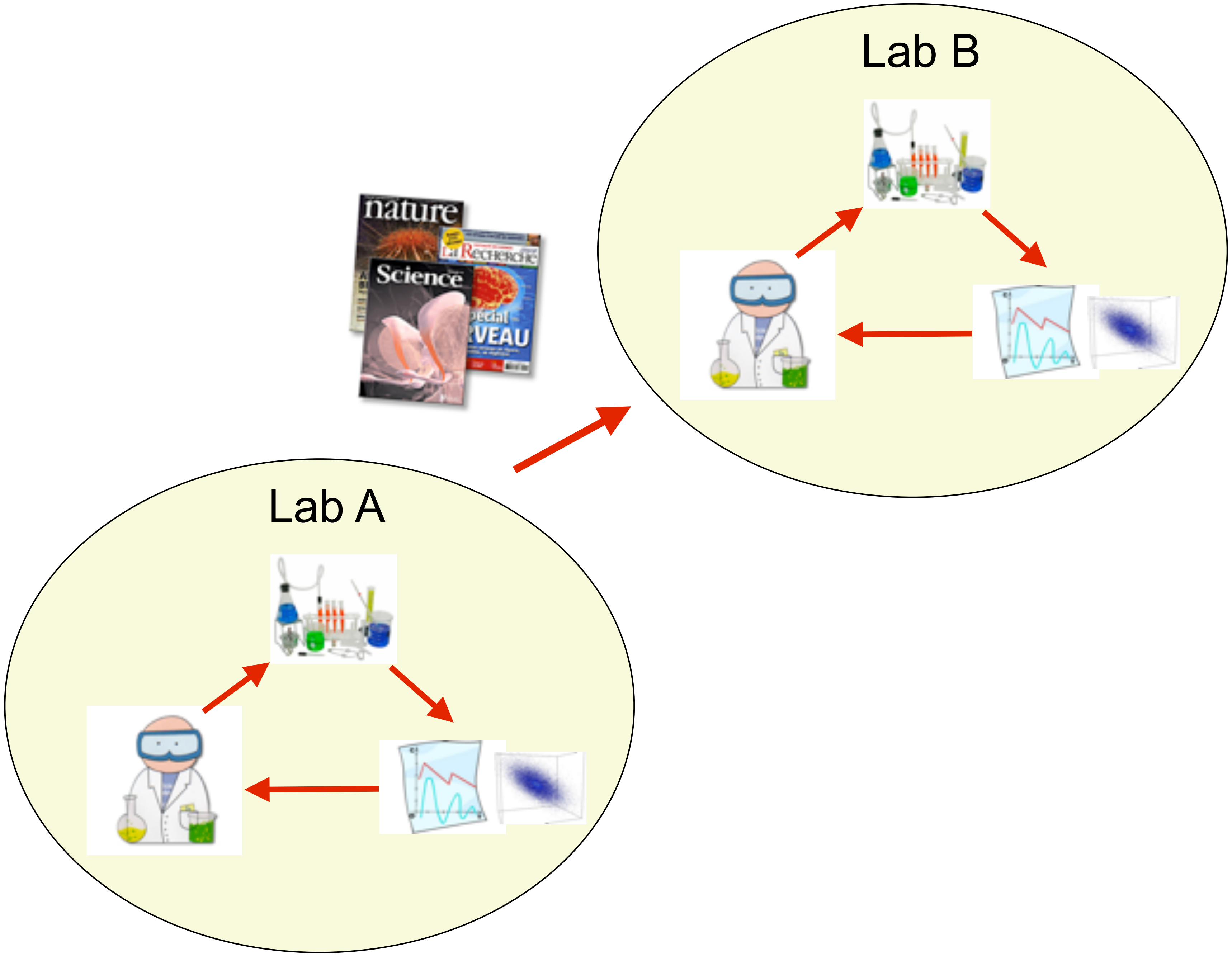

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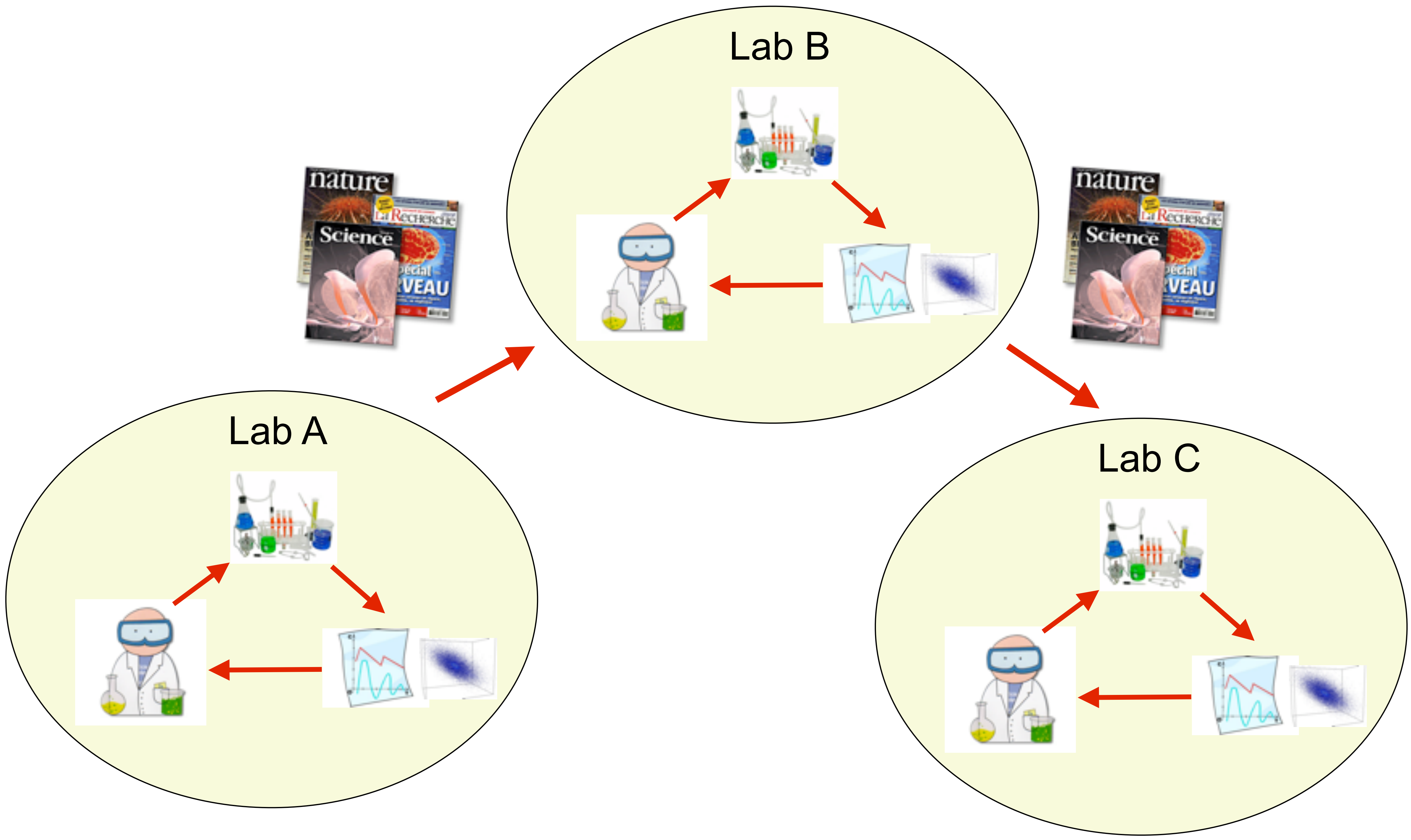

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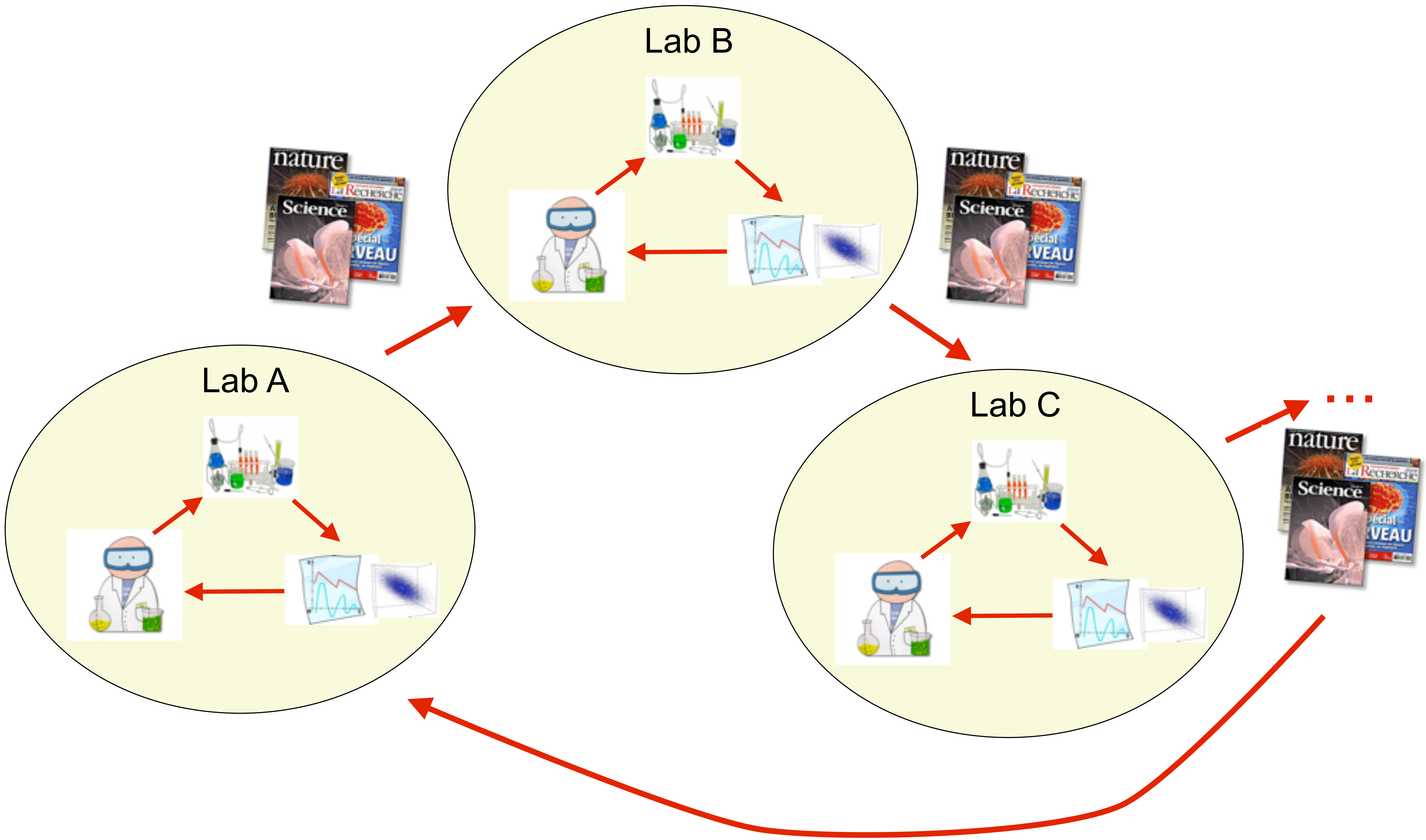

Motivation: Inferring Biological Pathways

Paul Alhquist
(Molecular Virology)

Audrey Gasch
(Genetics)

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virus

fruit fly

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13,071 single-gene
knock-down cell strains

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infect each strain
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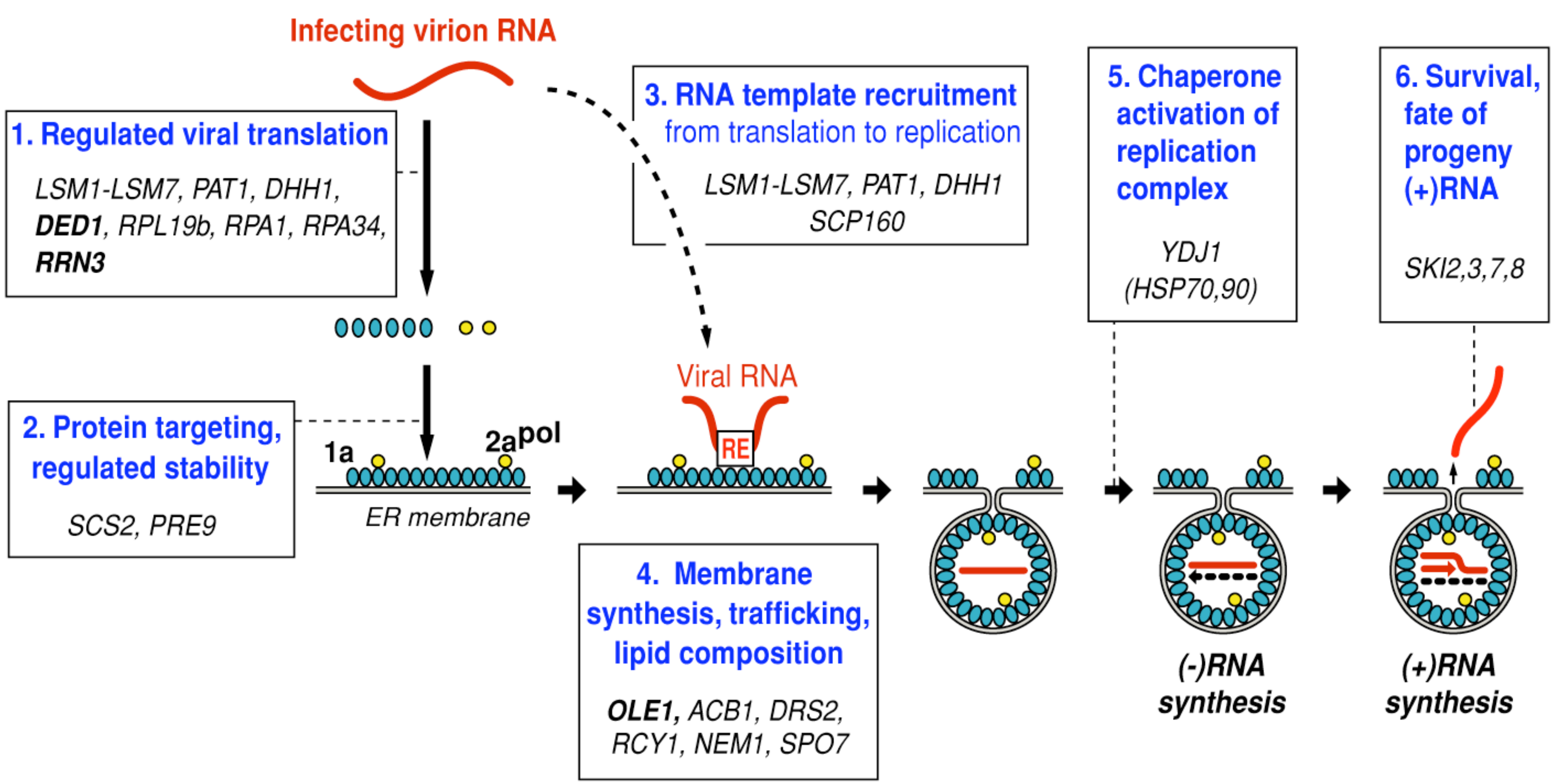

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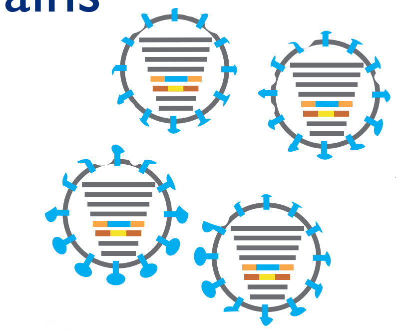

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**microwell
array**

"Drosophila RNAi screen identifies host genes important for influenza virus replication," Nature 2008. How do they confidently determine the ~100 out of 13K genes hijacked for virus replication from extremely noisy data?

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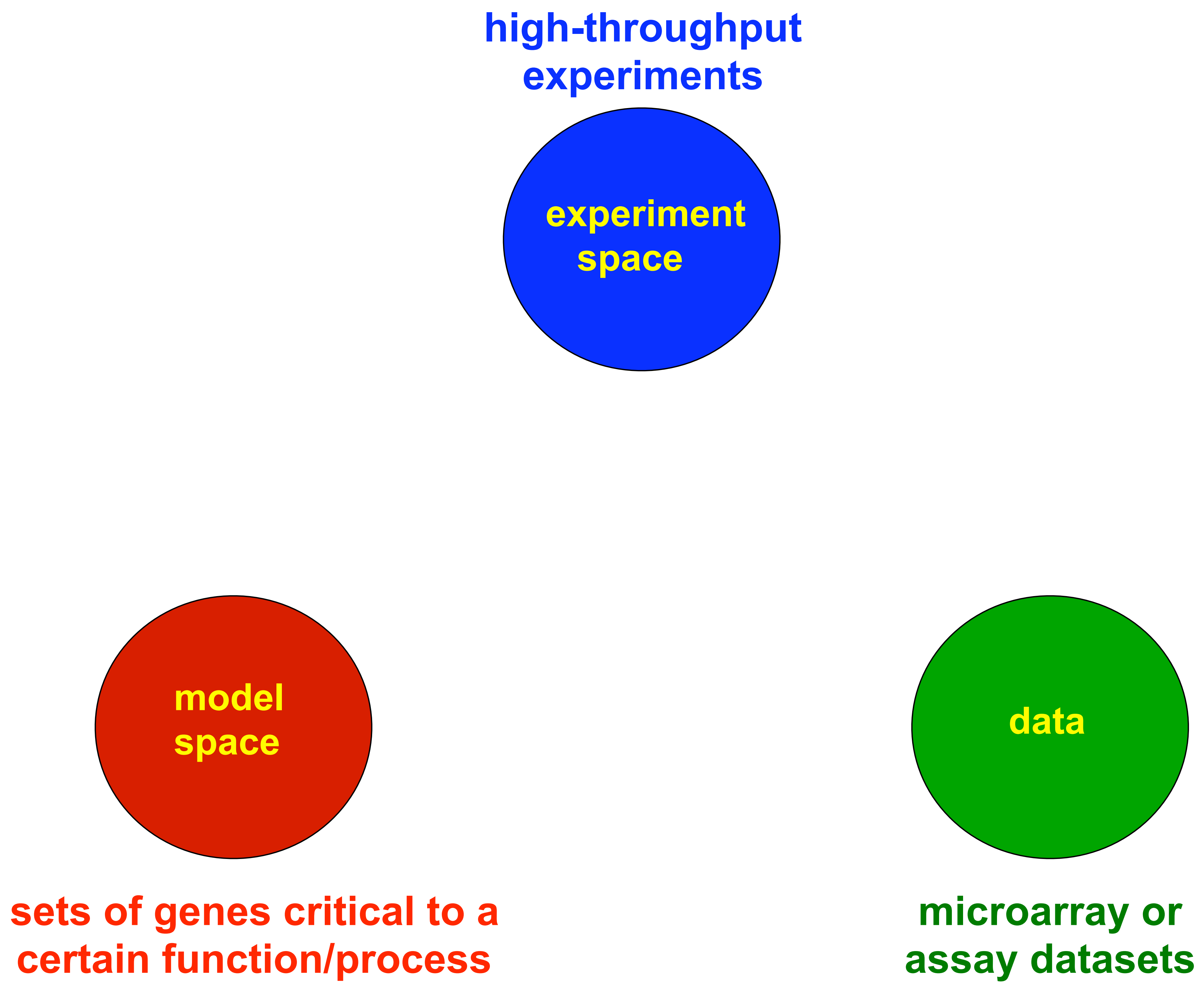
vastly more efficient that replicating all 13K experiments many times

Feedback from Data Analysis to Data Collection



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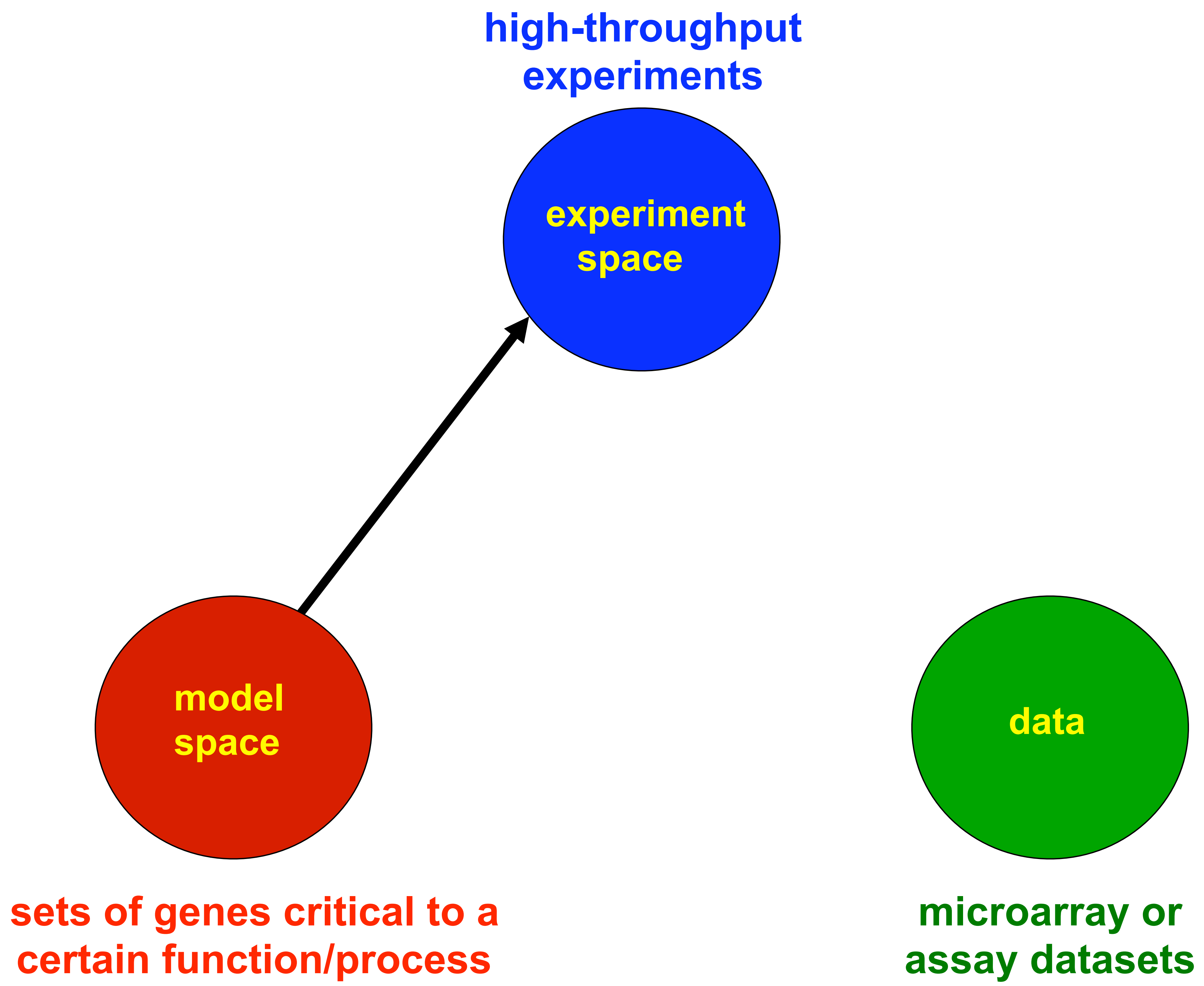

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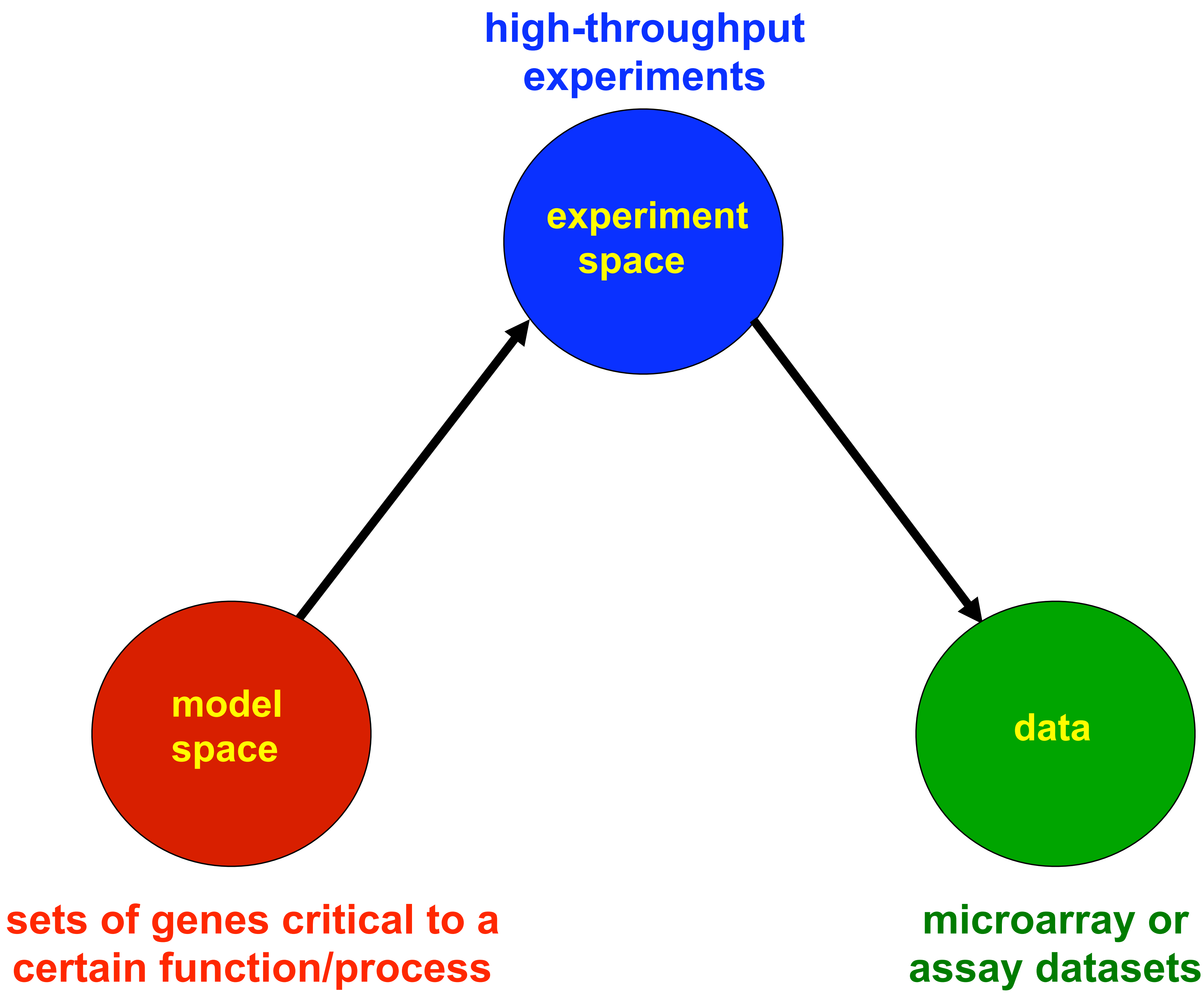

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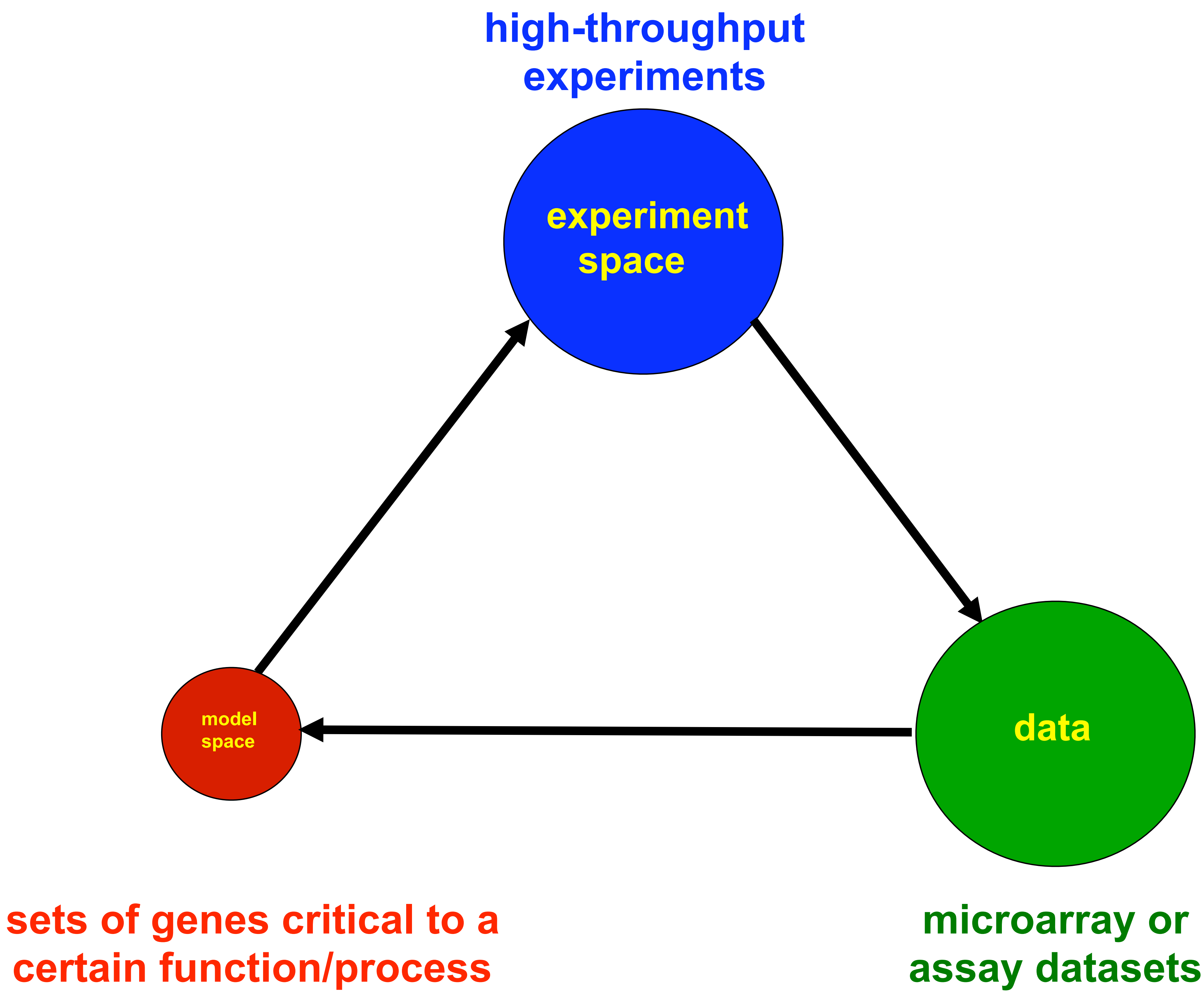
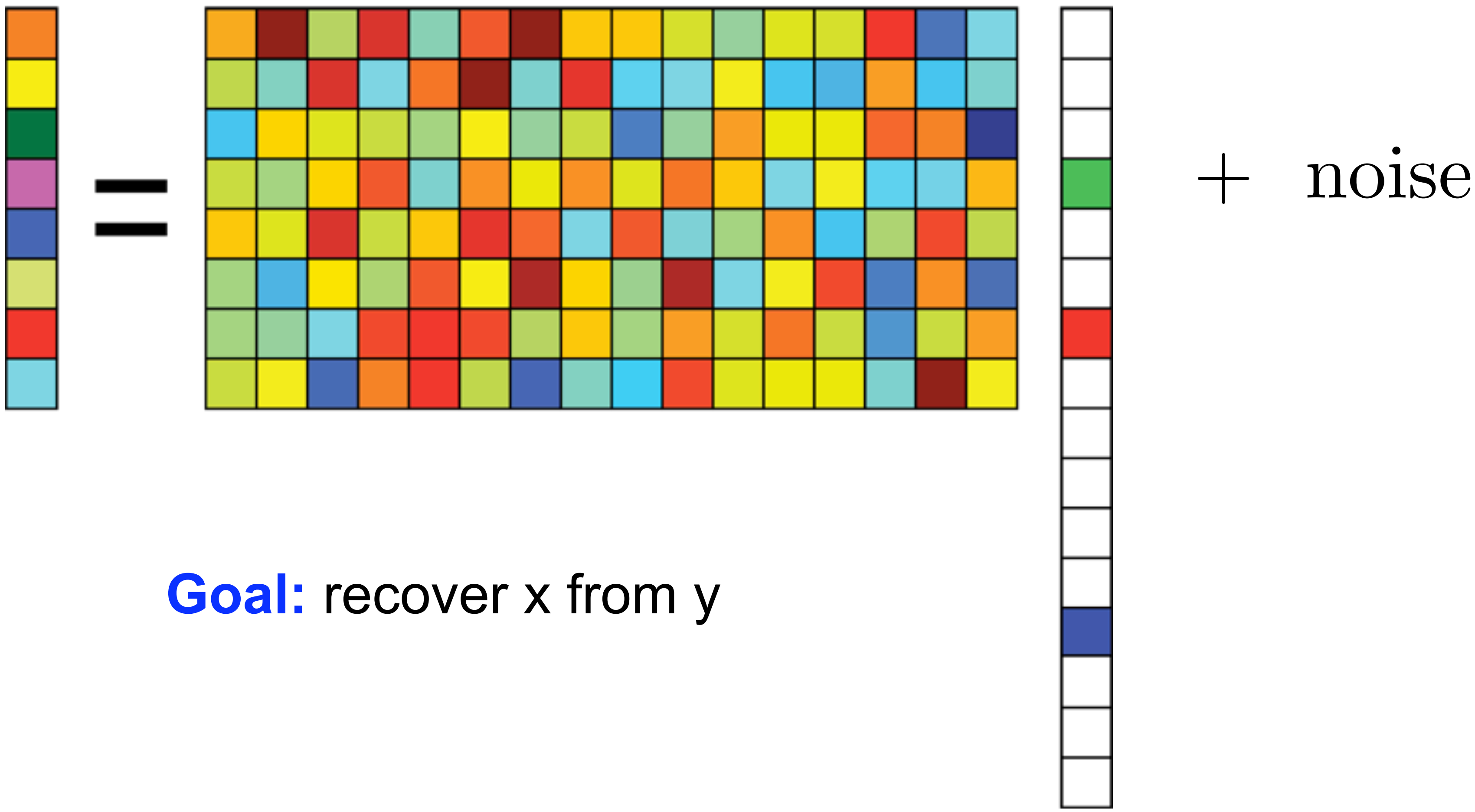


Adaptive Sensing for Sparse Recovery

(image reconstruction, compressed sensing, inverse problems)

$$y \;=\; A\,x \;+\; w \;, \text{ with } A \in \mathbb{R}^{m\times n},\; x \in \mathbb{R}^n \text{ (but sparse)},\; w \sim \mathcal{N}(0, I)$$



Goal: recover x from y

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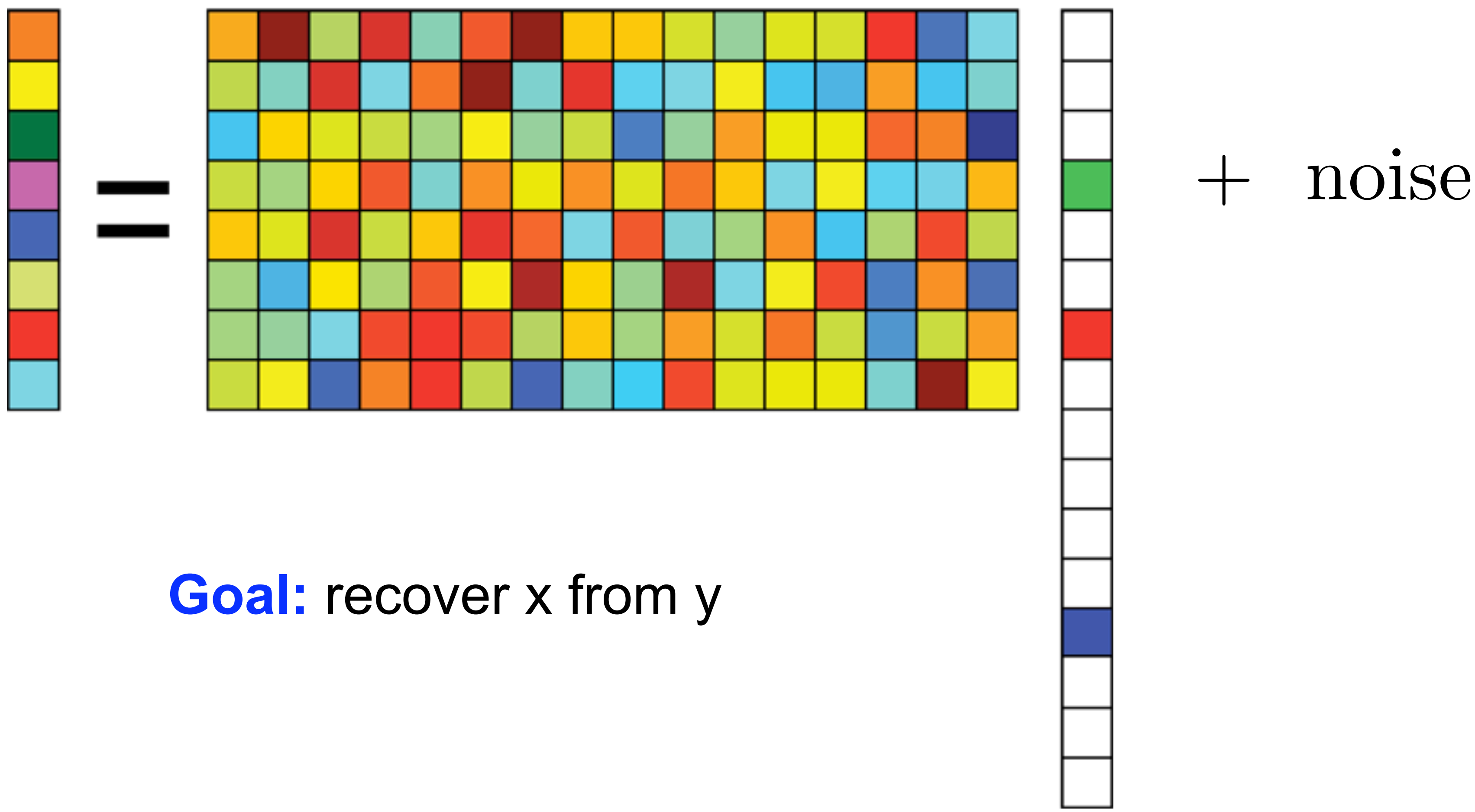


Goal: recover x from y

Is sequentially designing (rows of) A advantageous ?

Motivation: Randomized Experiments

$\widetilde{y}$ =  $\times$  $+$ 

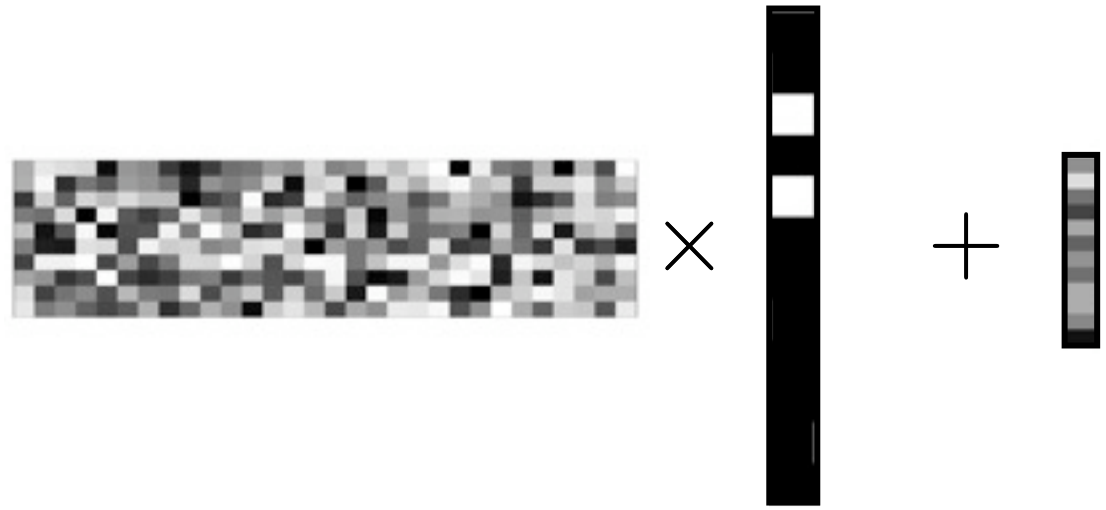
$k \times 1$

sparse signal

noise

indirect (randomized) measurement

Motivation: Randomized Experiments

$\widetilde{y} =$ 

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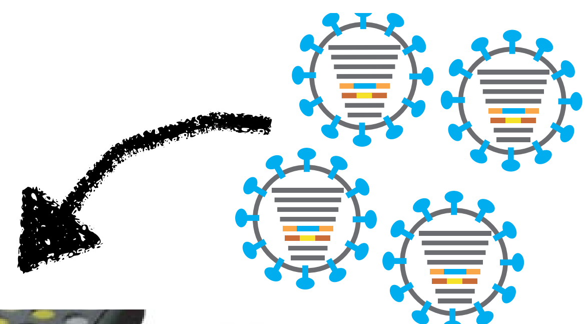

put mixtures of single-deletion strains into each well

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A visual representation of the math behind compressive sensing

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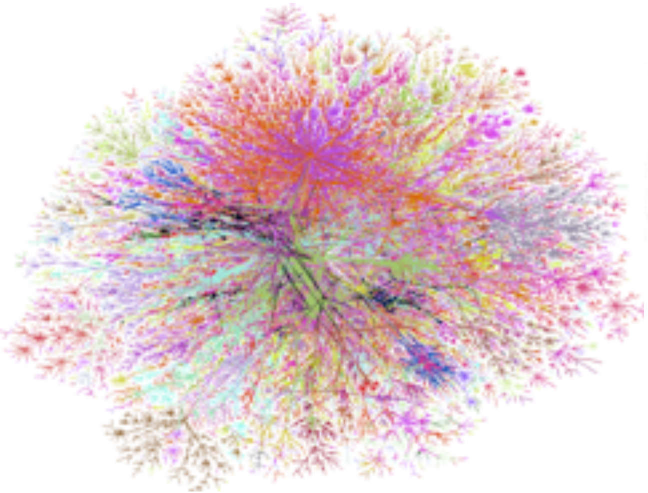
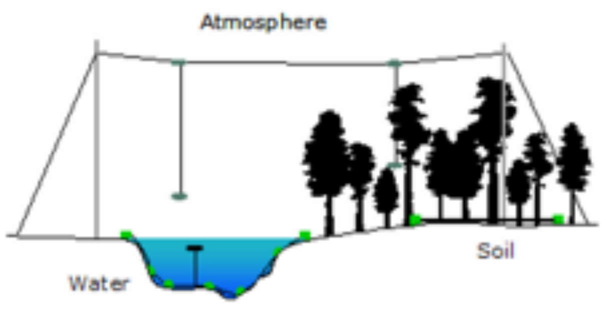
Sensing and Inference in Large Networked Systems

Sensing and Inference in Large Networked Systems

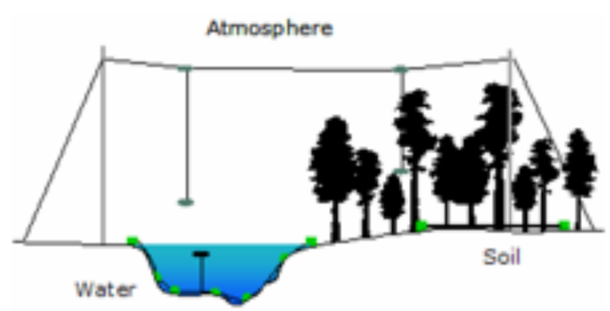




Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)

Sensing and Inference in Large Networked Systems





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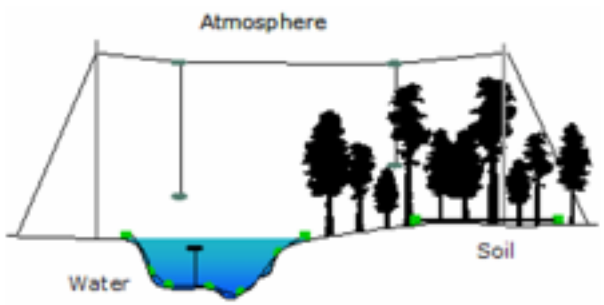
Social Networks

Sensing and Inference in Large Networked Systems



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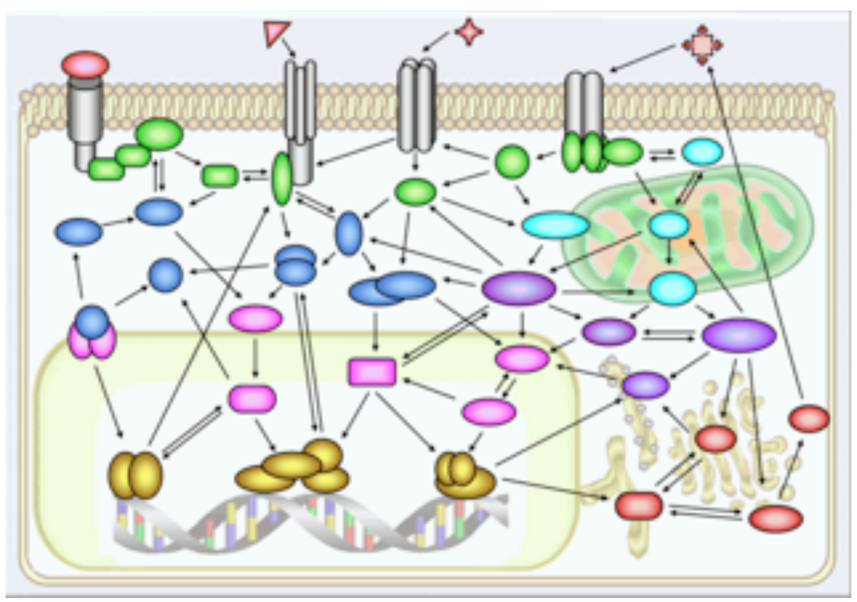
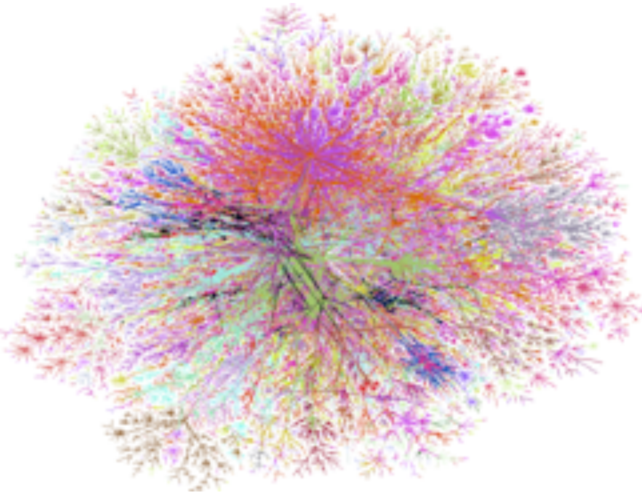
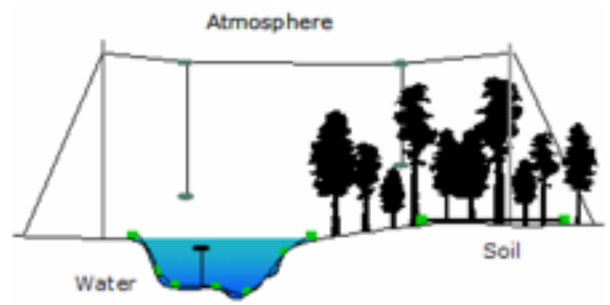


Biological Networks
(JMDBase)

Sensing and Inference in Large Networked Systems





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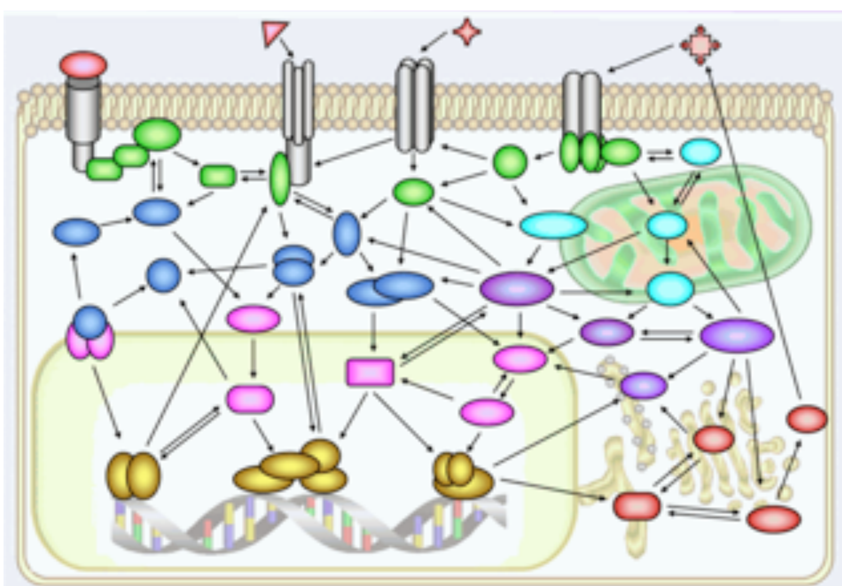

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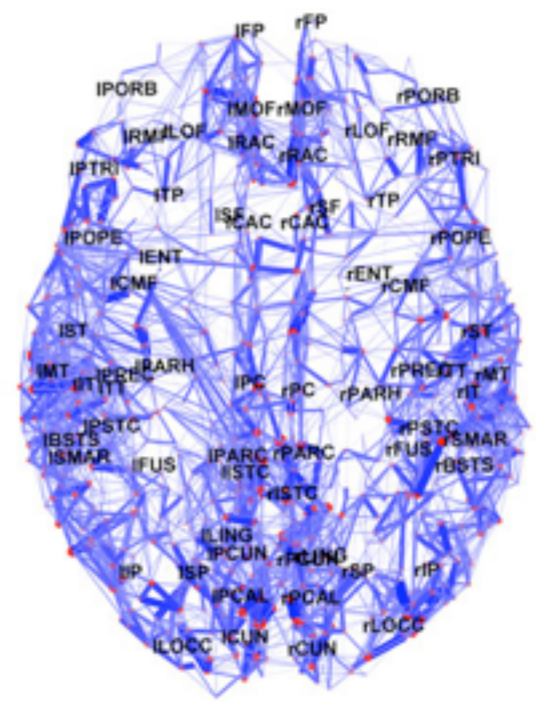

Brain Networks
(Worsley et al, 2005)

Sensing and Inference in Large Networked Systems

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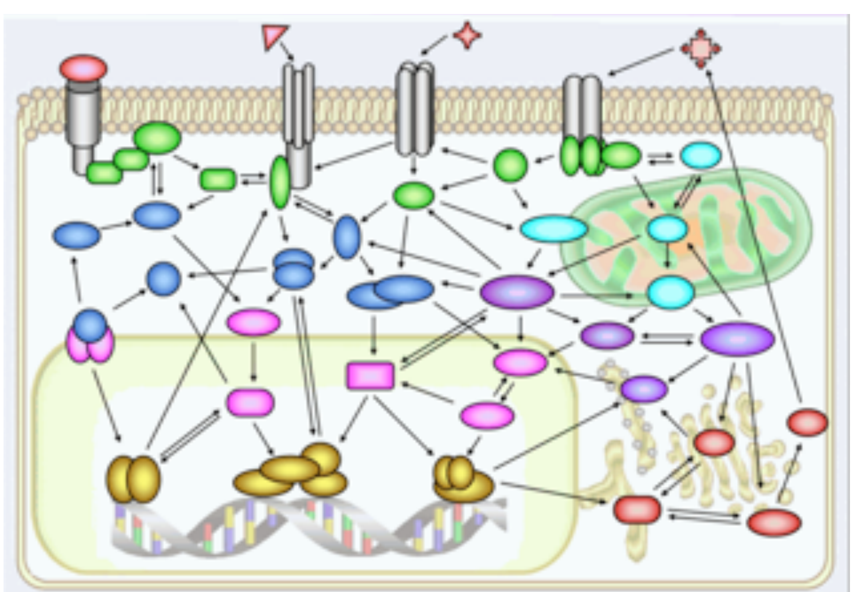

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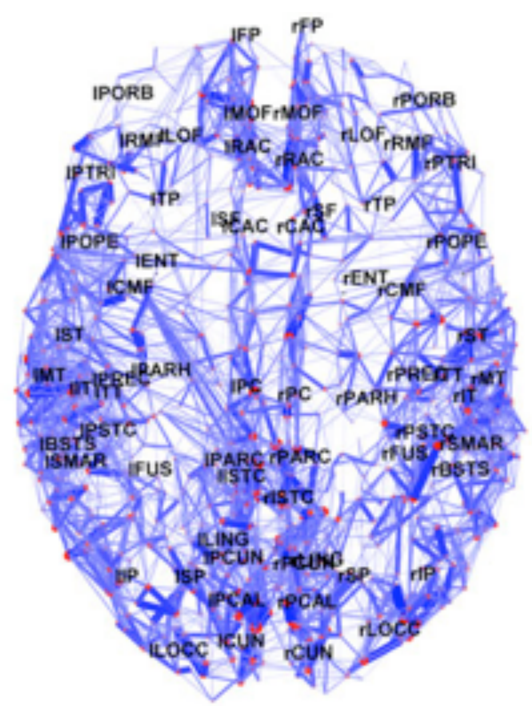


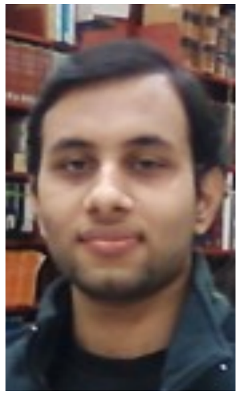
Brain Networks
(Worsley et al, 2005)

Challenges:

- Inferring structure & function of the system
- Optimized design & resource allocation
- Pattern analysis & anomaly detection

Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.


Gautam Dasarathy


Brian Eriksson

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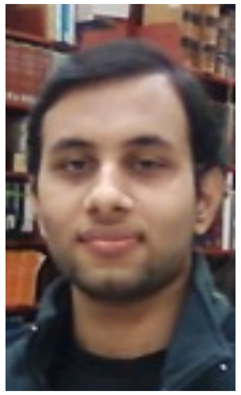
Network(s) of interactions can be revealed via **clustering** based on measured features



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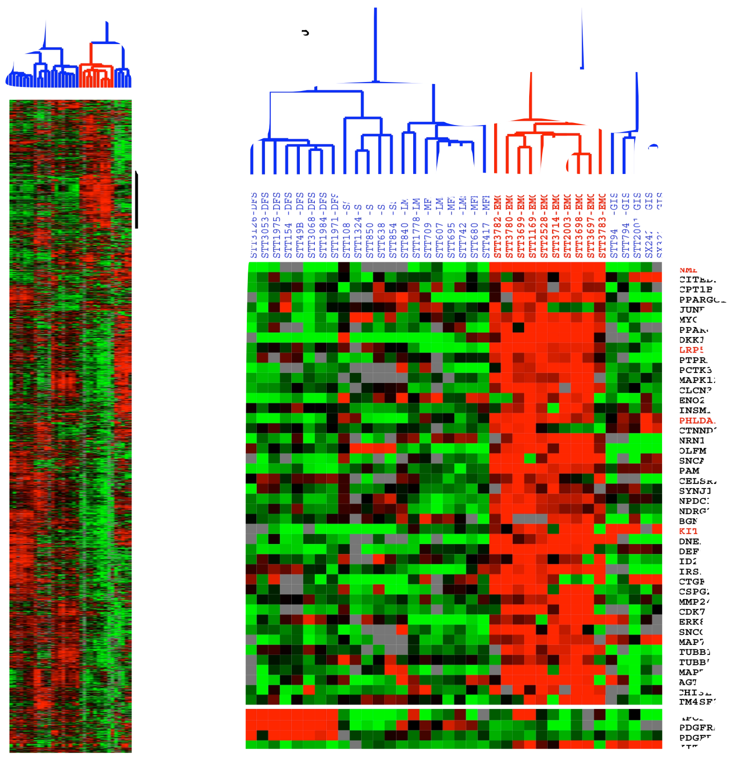
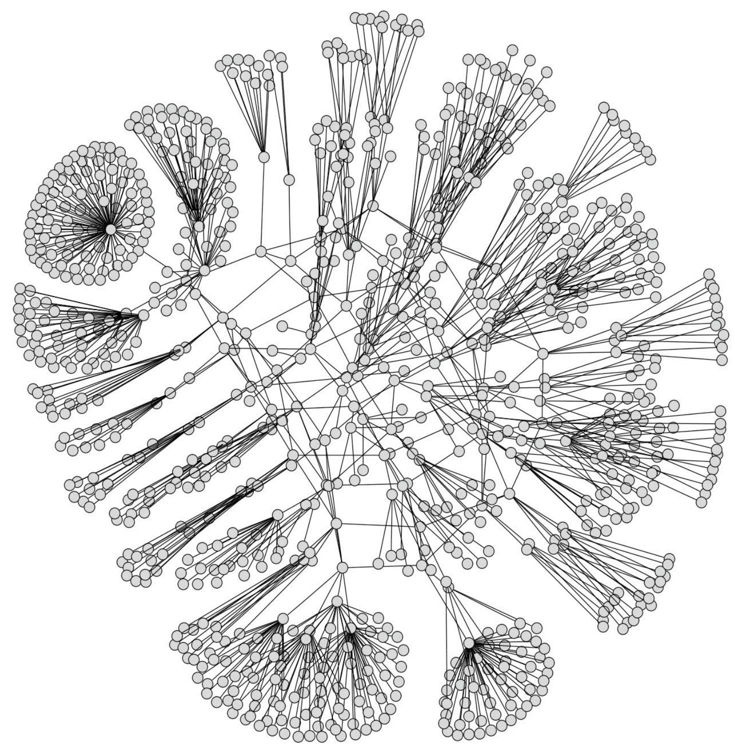
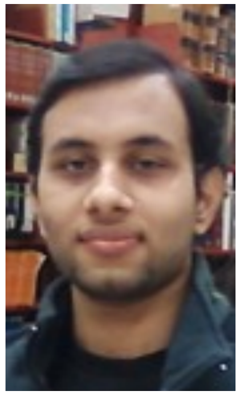


genes and expression/
interaction profiles



network routers and
traffic/distance profiles



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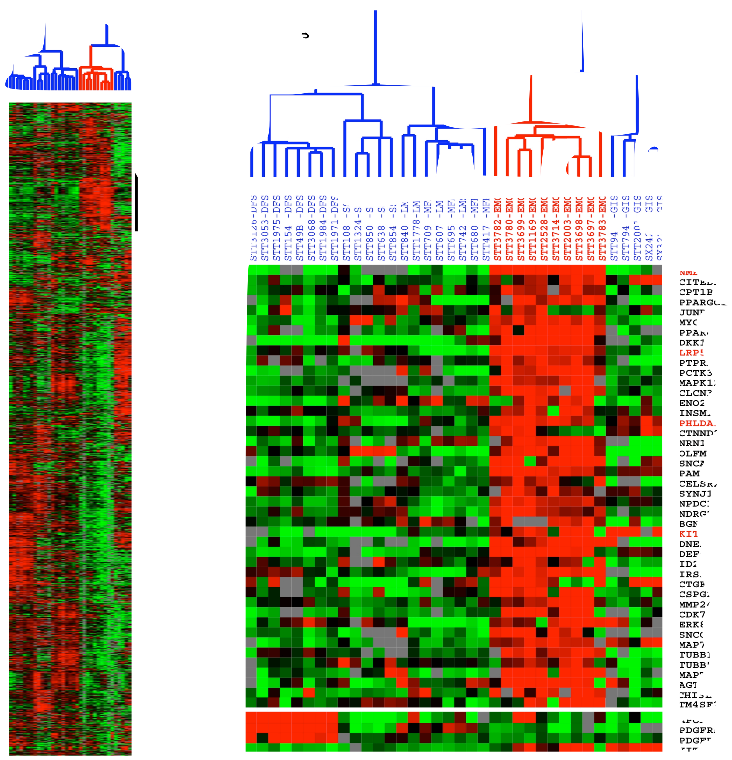

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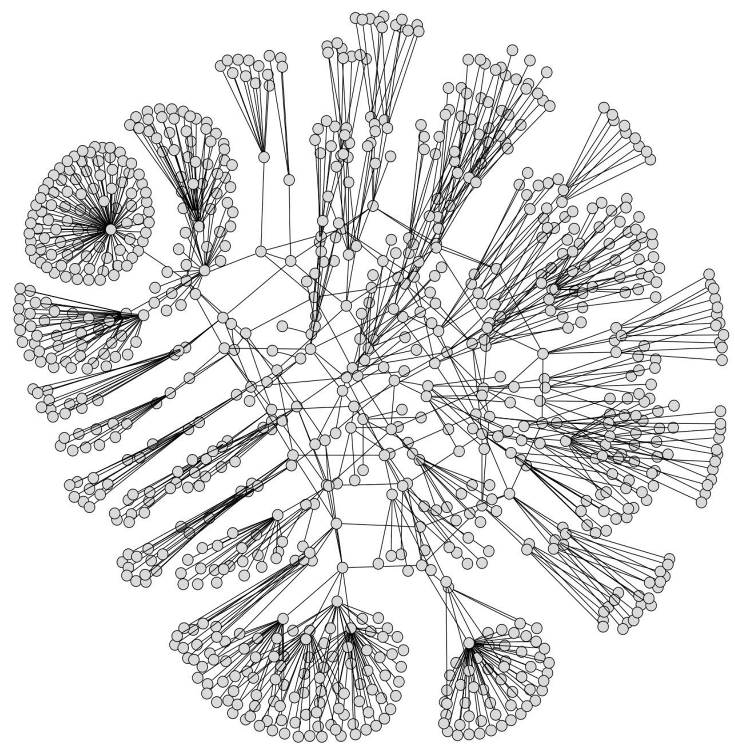

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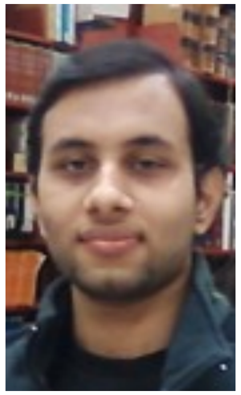

Gautam Dasarathy

Brian Eriksson

Similarity-Based Clustering: Each component (gene/router) has an associated feature (measurement profile). Components can be clustered based on feature similarities.

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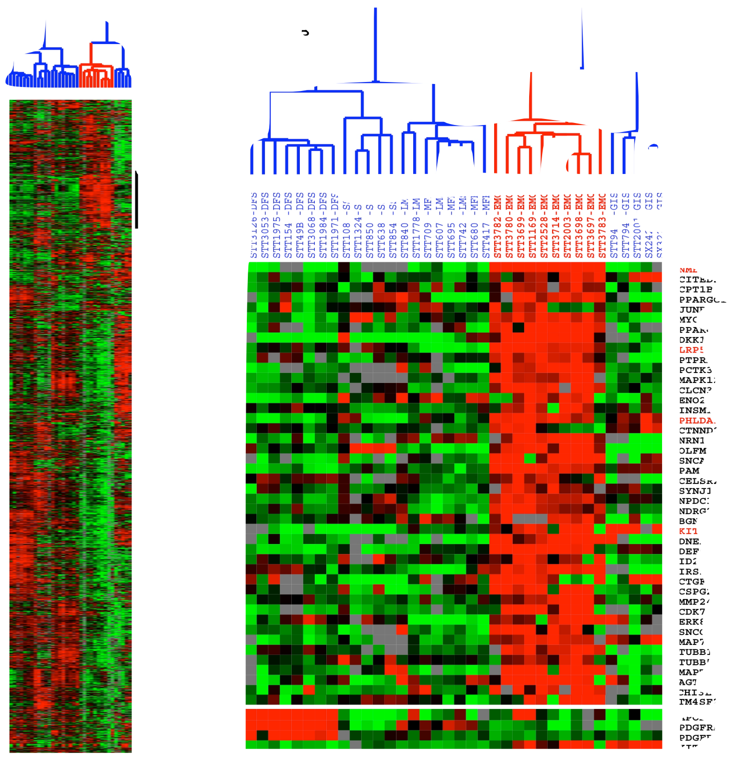

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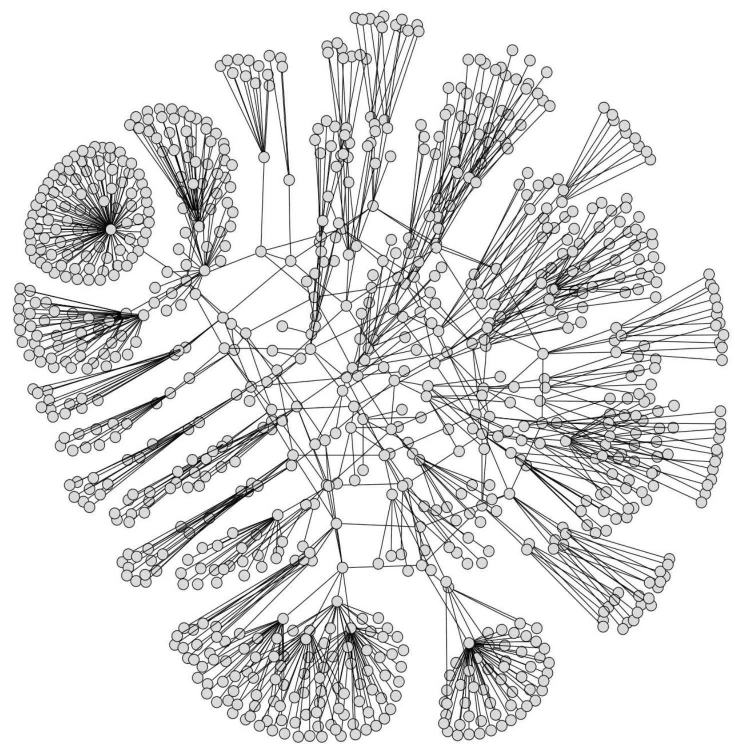


network routers and
traffic/distance profiles

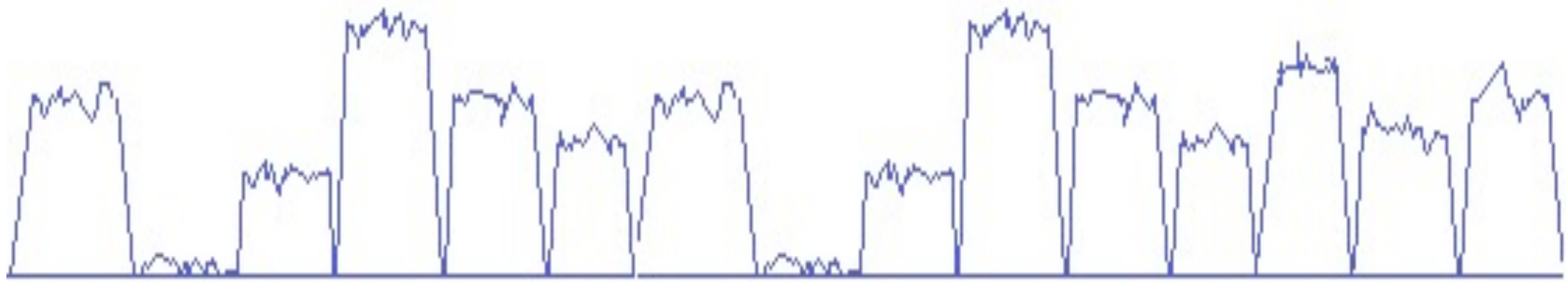
Gautam Dasarathy　Brian Eriksson

Similarity-Based Clustering: Each component (gene/router) has an associated feature (measurement profile). Components can be clustered based on feature similarities.

Recent Result: A sequential method for selecting "informative" similarities that produces accurate clusters from as few as $3N \log N$ similarities.

Cognitive Radio Spectrum Sensing

"primary" users have preference over "secondary" users



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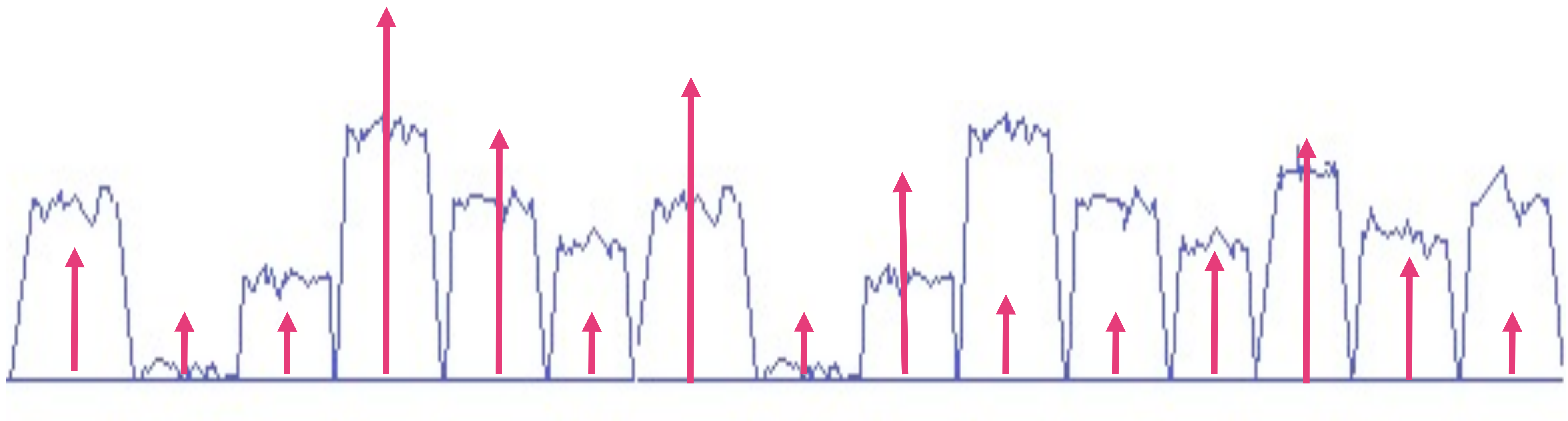

most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users "sense" spectrum to find an unoccupied channel

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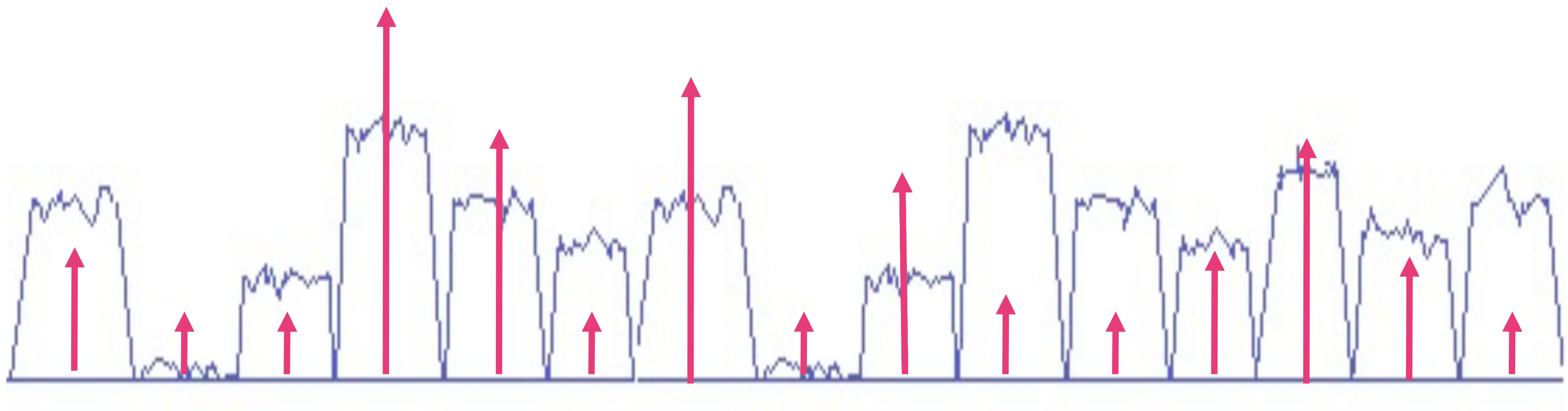


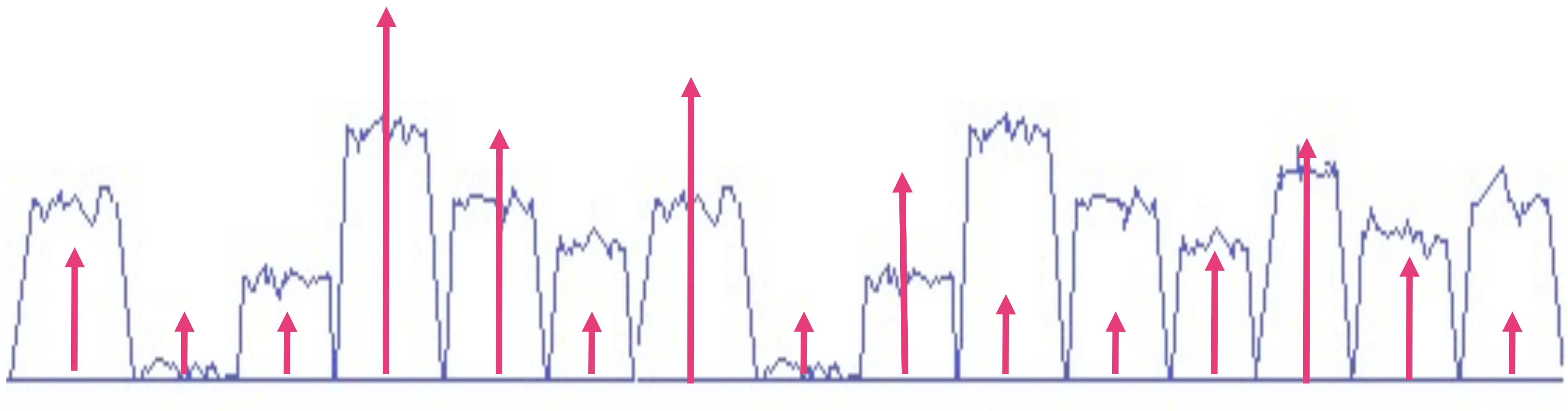
most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users "sense" spectrum to find an unoccupied channel

Goal: Find open channel(s) as quickly as possible. Two approaches:

1) listen to each channel for a fixed amount of time and make decision
2) listen to each channel for a data-adaptive amount of time to make decisions as quickly as possible

Cognitive Radio Spectrum Sensing

"primary" users have preference over "secondary" users



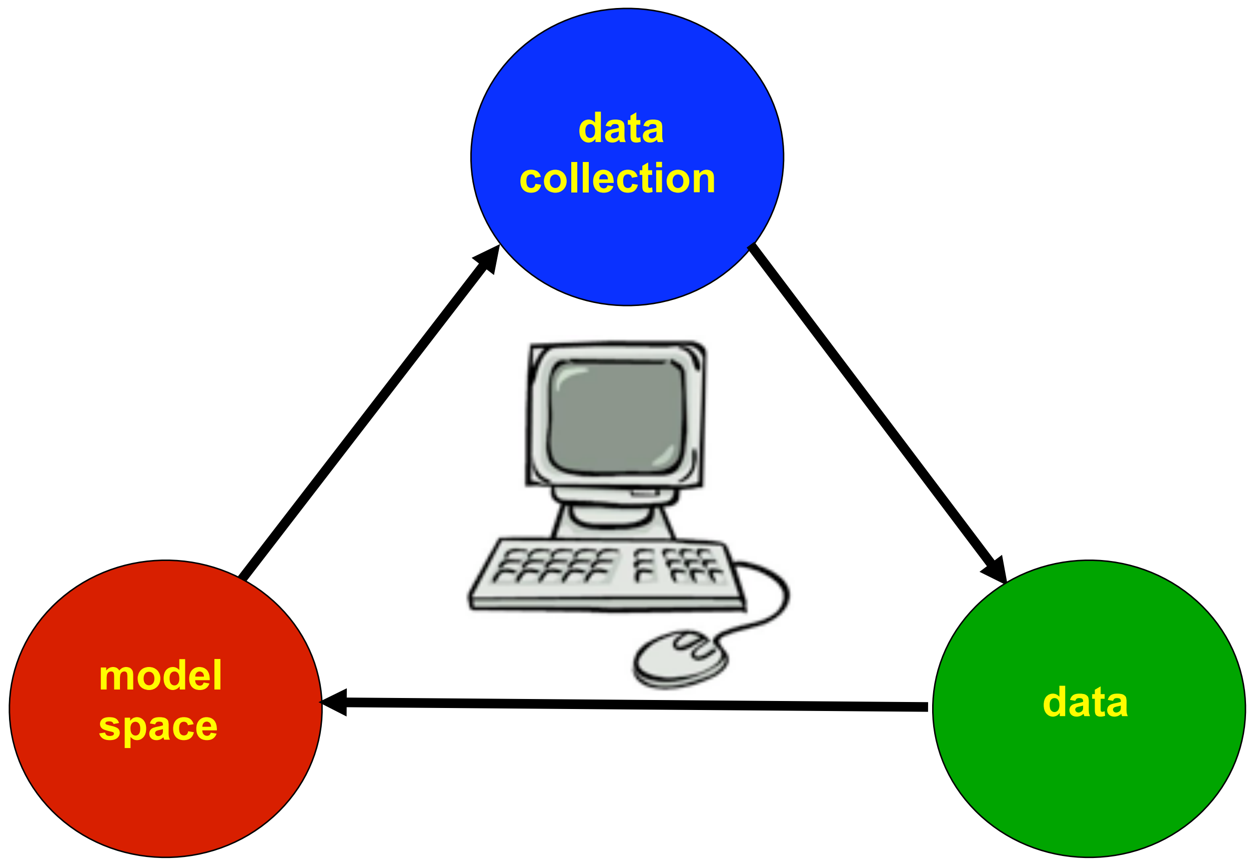
most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users "sense" spectrum to find an unoccupied channel

Goal: Find open channel(s) as quickly as possible. Two approaches:

1) listen to each channel for a fixed amount of time and make decision
2) listen to each channel for a data-adaptive amount of time to make decisions as quickly as possible

adaptive spectrum sensing is significantly more time-efficient than fixed sensing

Active Learning in Machines and Humans



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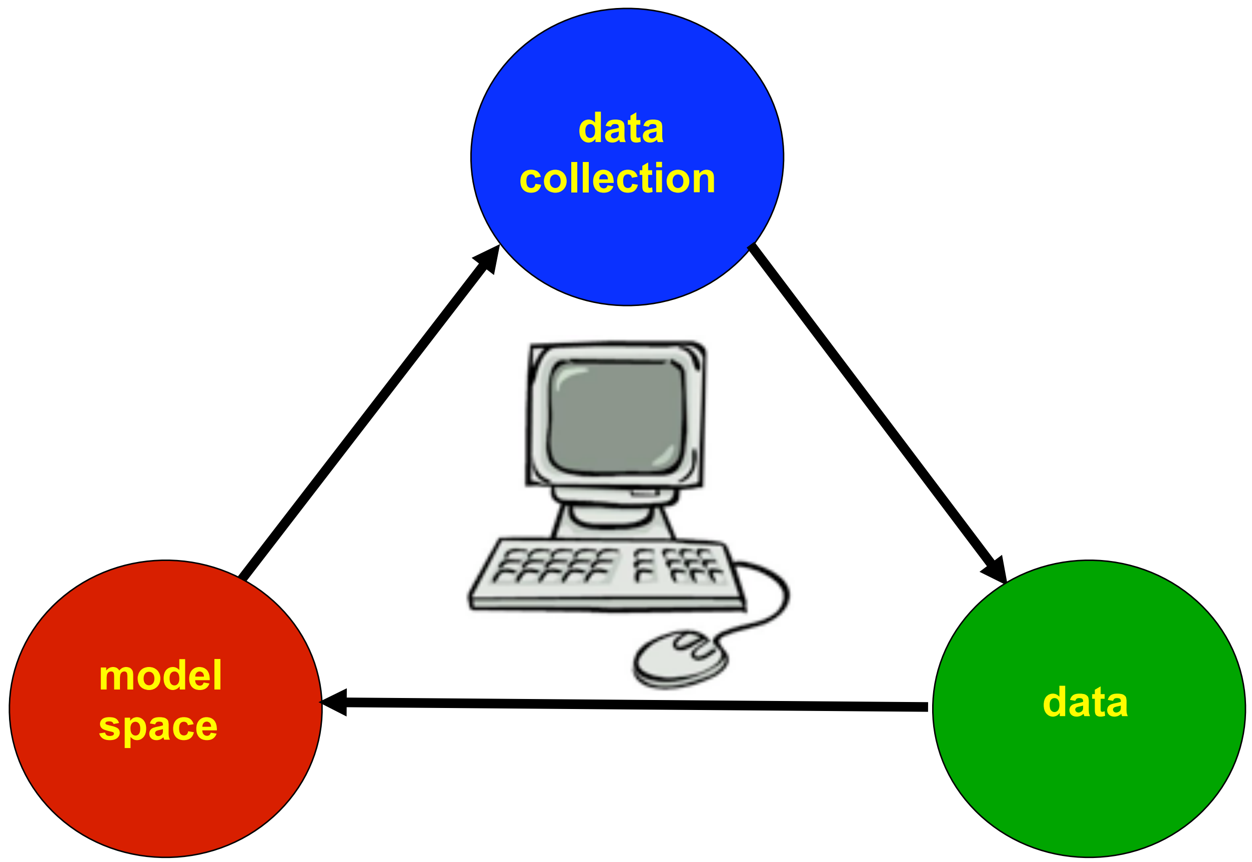

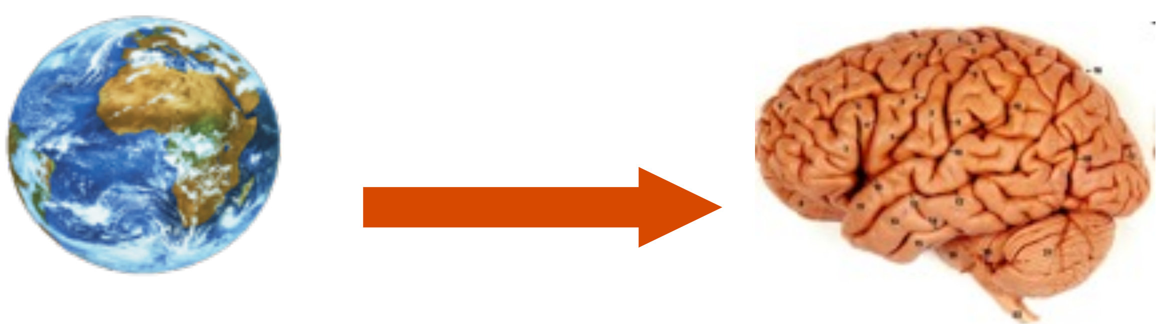

Sensing

Computing

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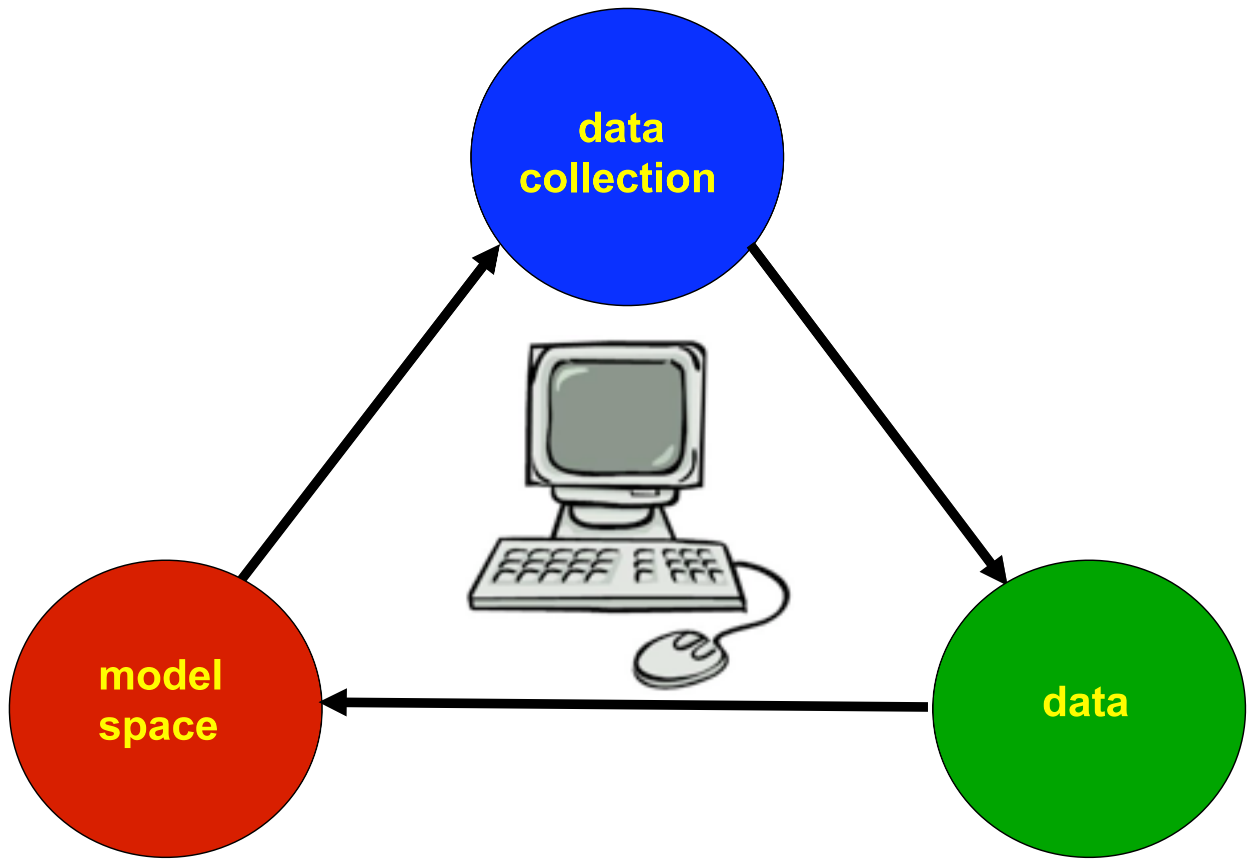
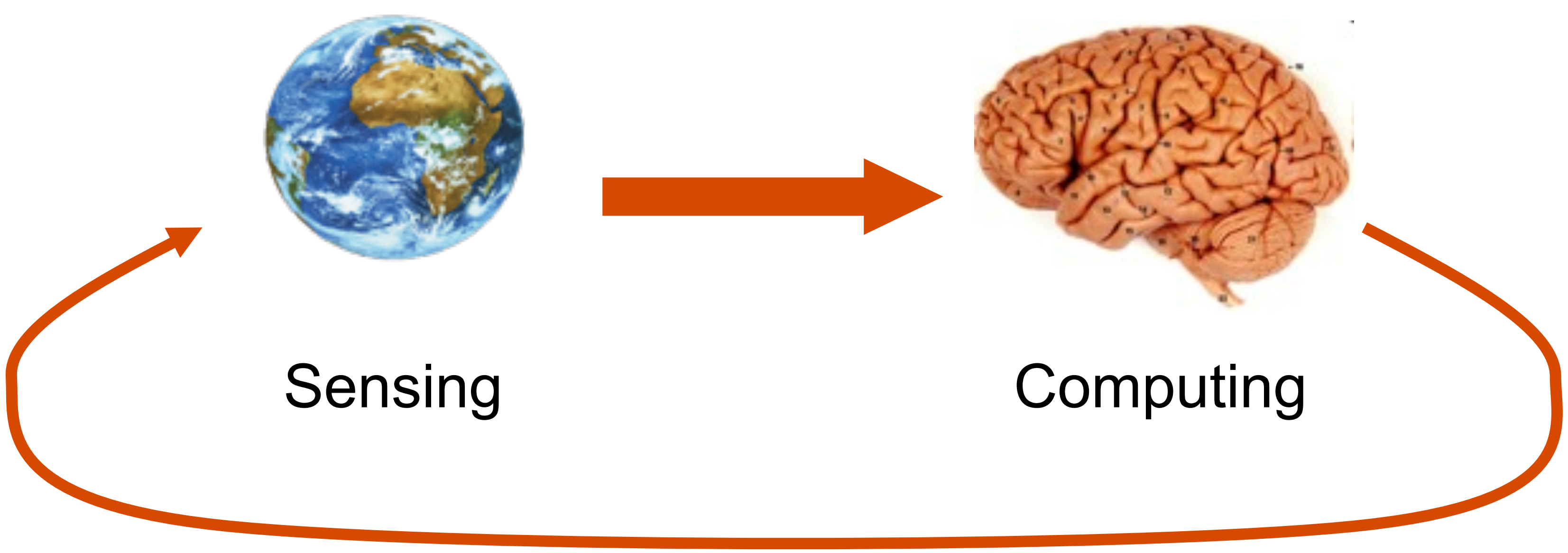
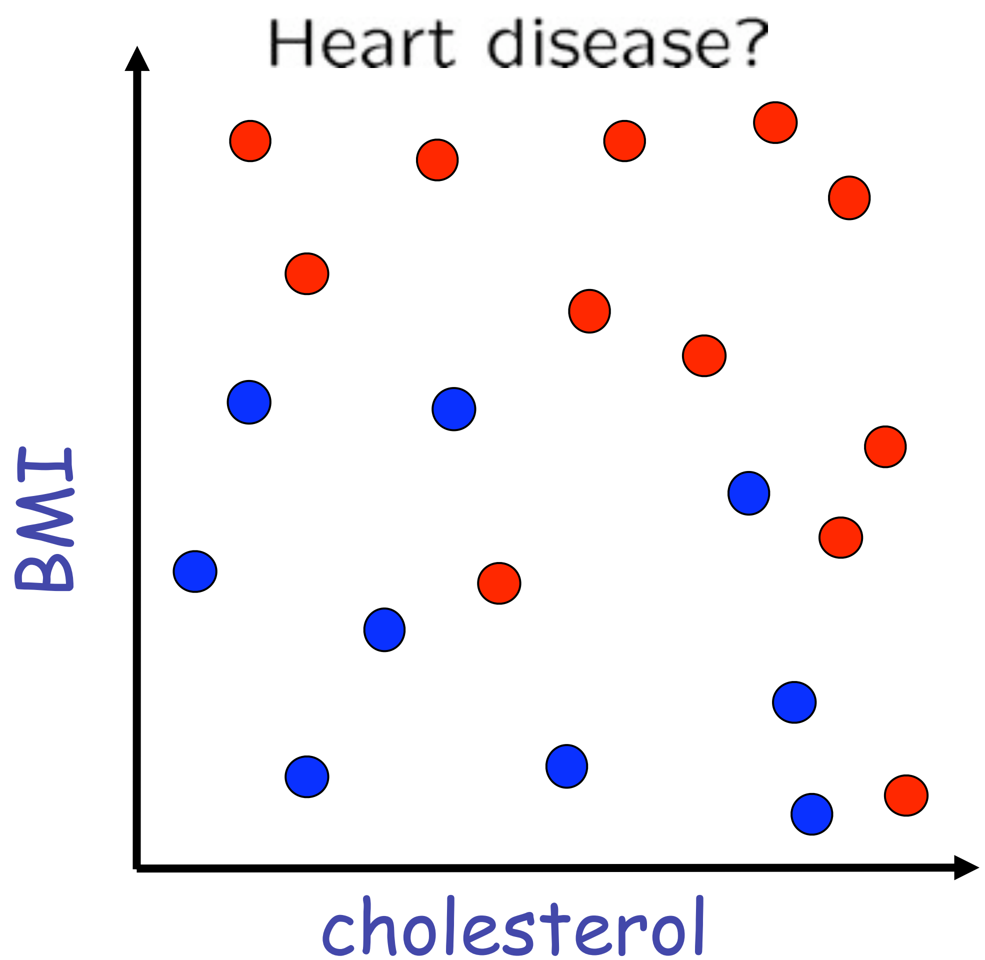




Active Learning

Learn to predict labels y from features x based on training examples $\{(x_i, y_i)\}_{i=1}^n$



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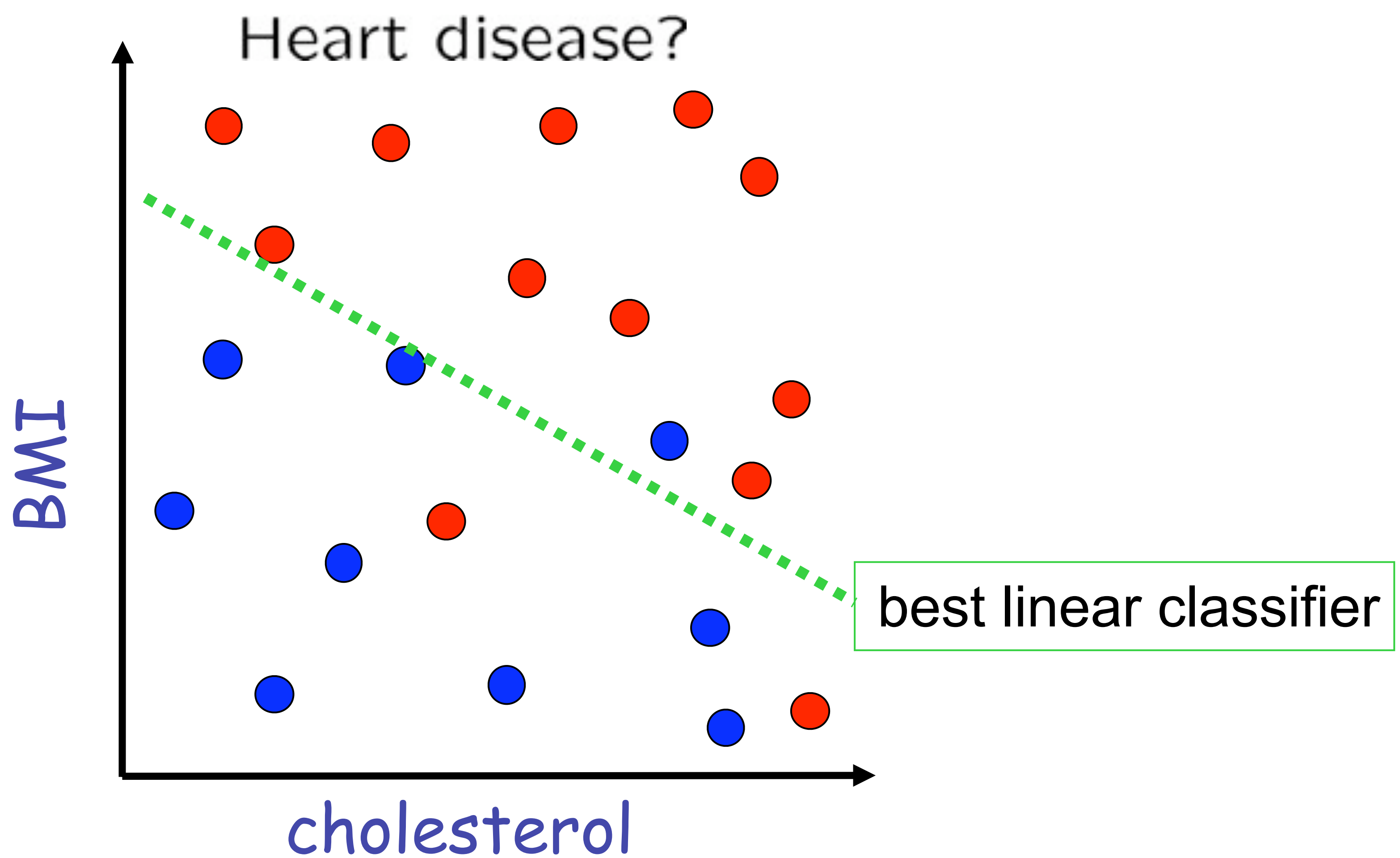

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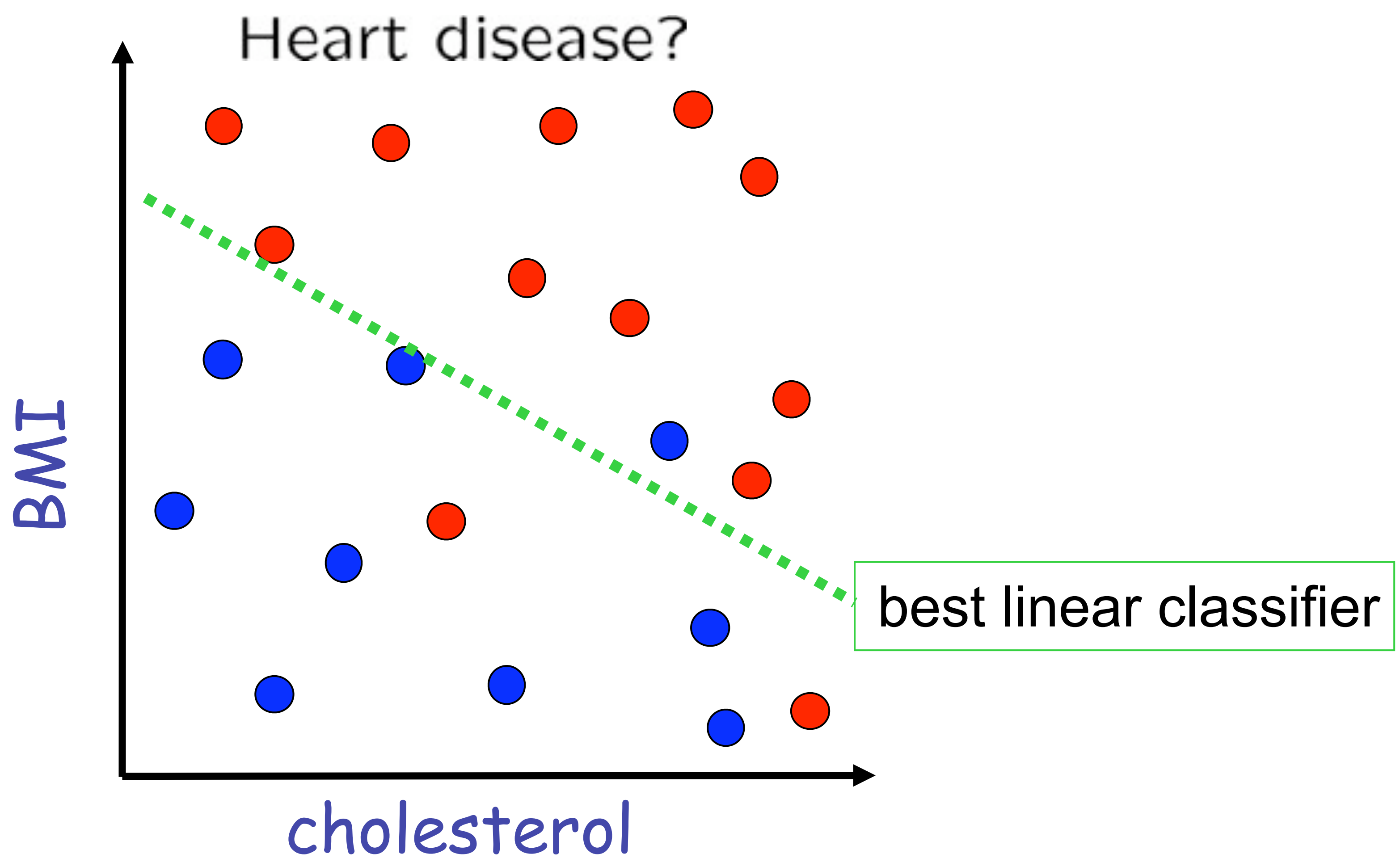

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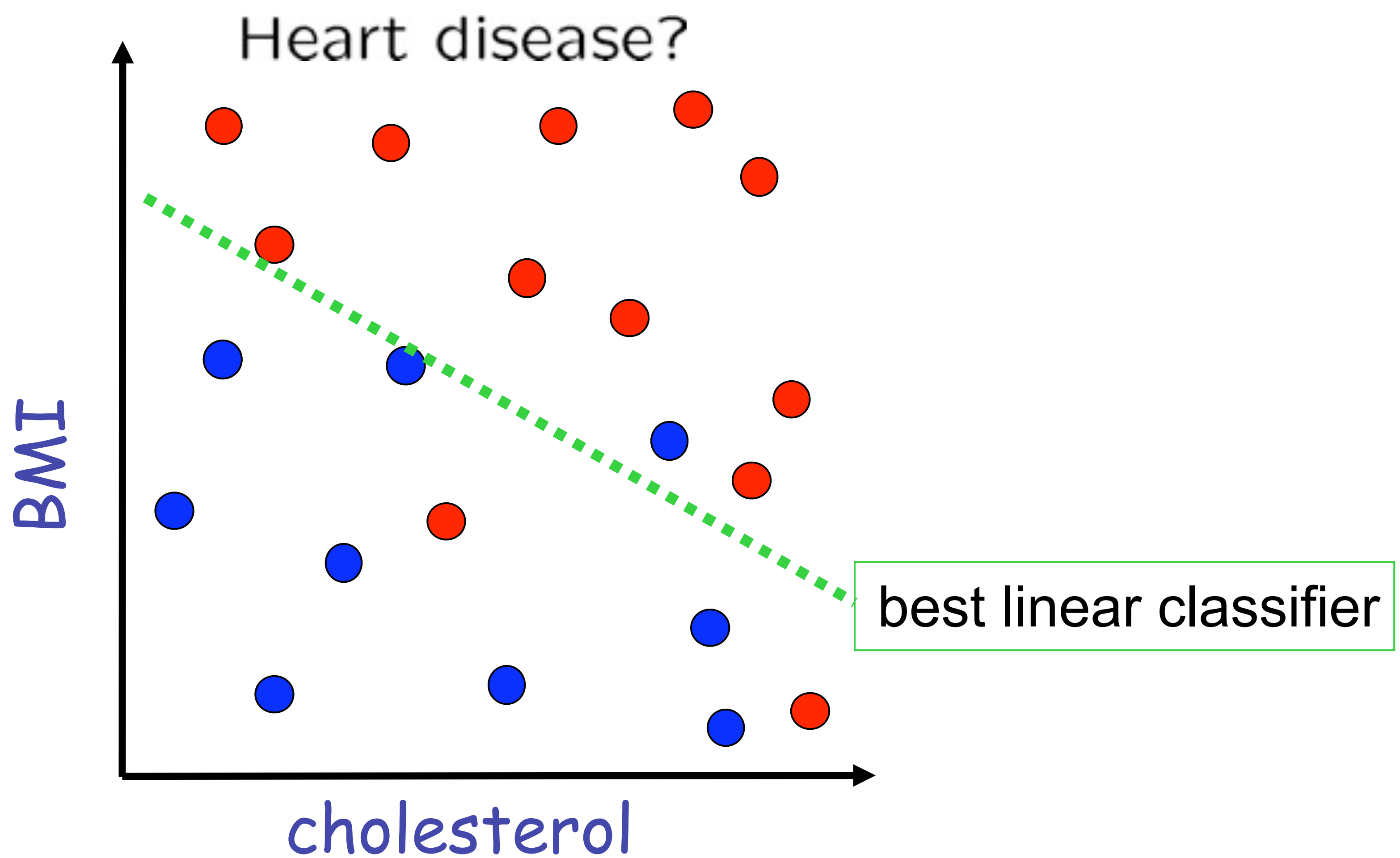

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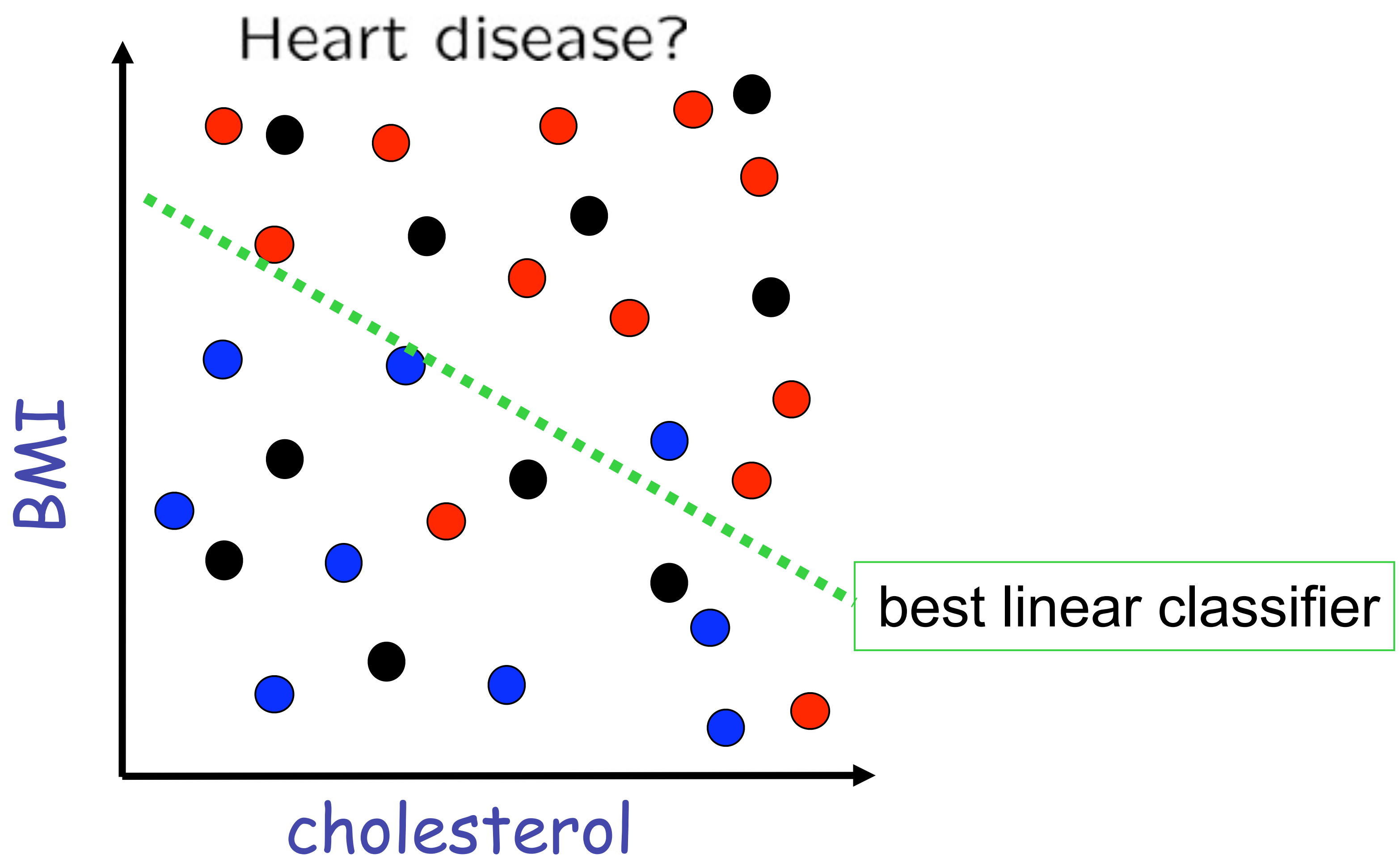

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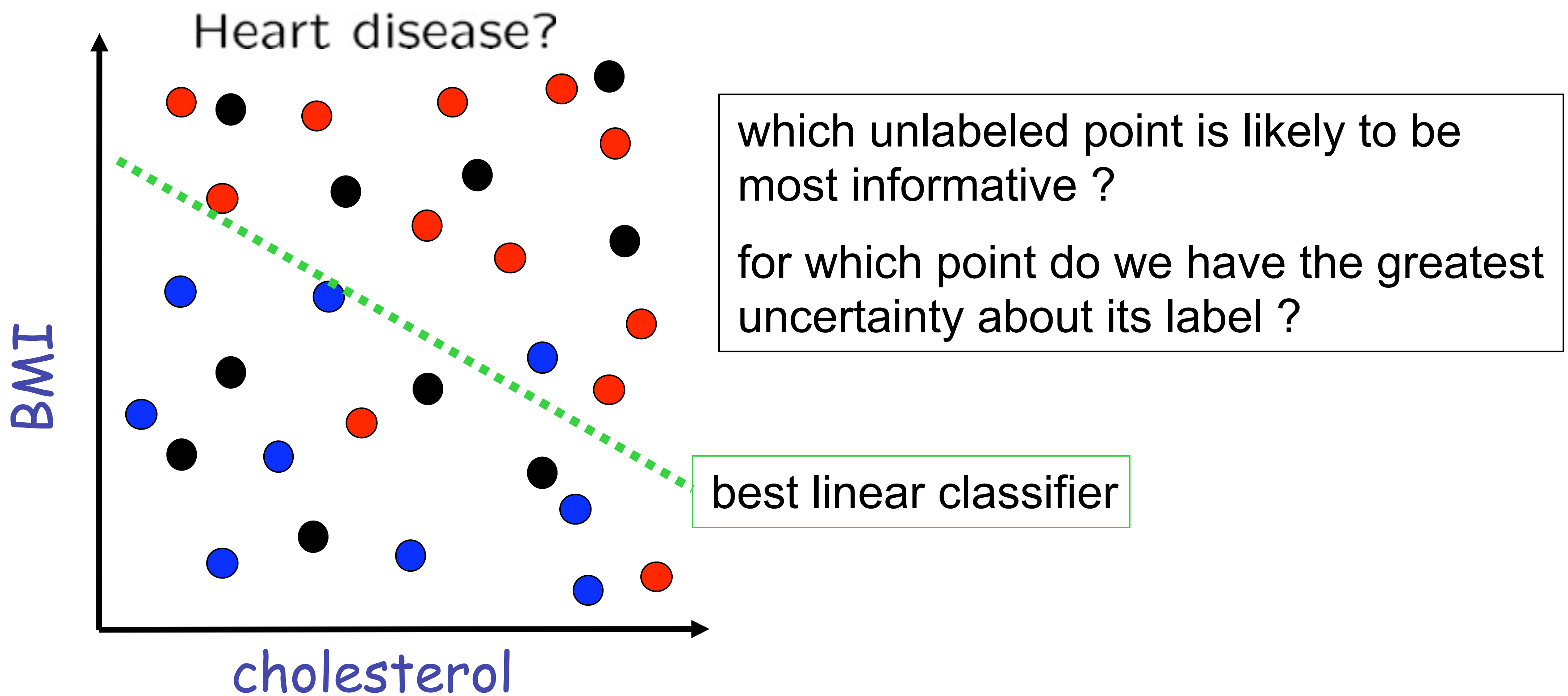

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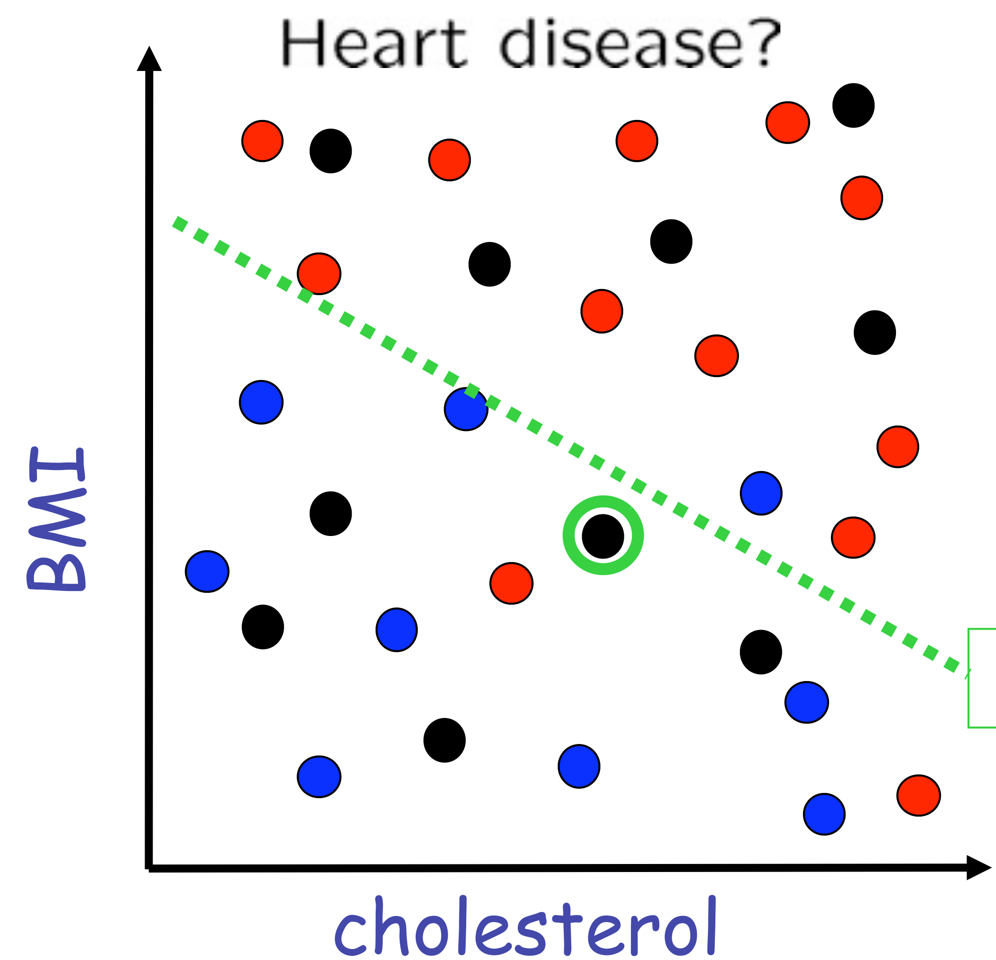

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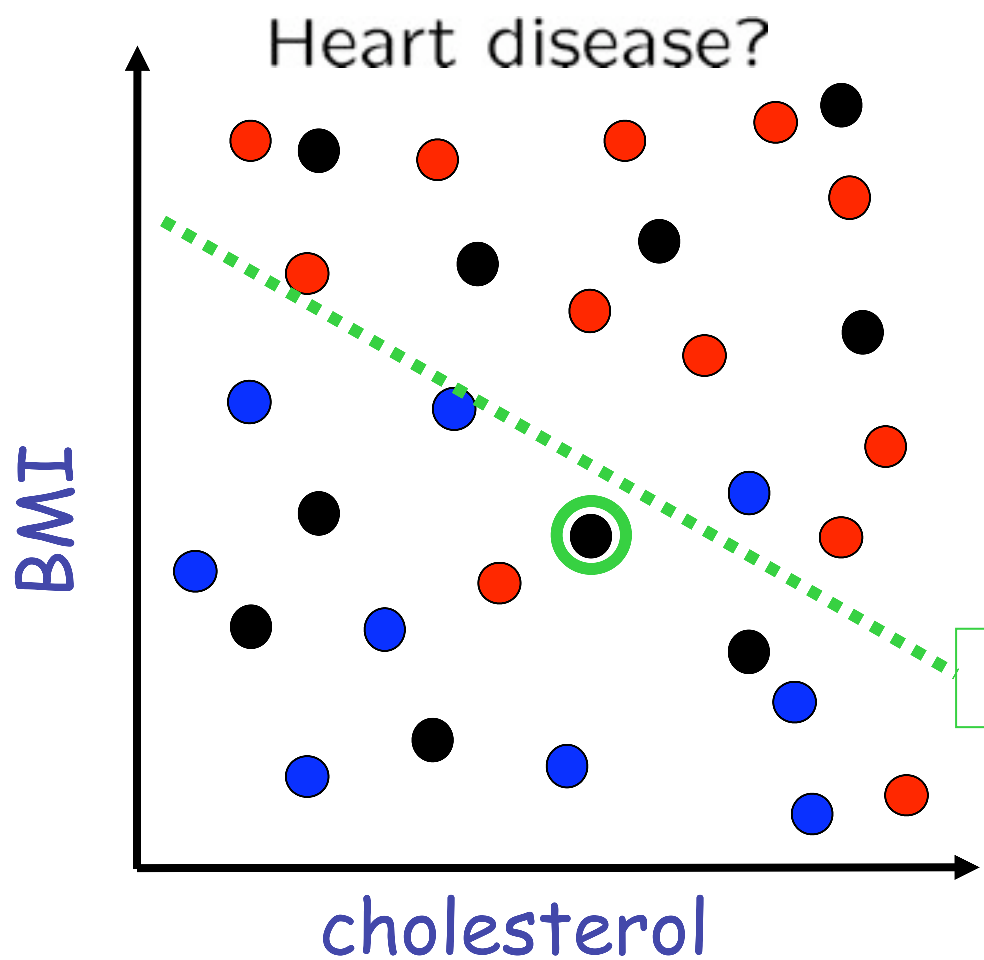


which unlabeled point is likely to be most informative ?

for which point do we have the greatest uncertainty about its label ?

best linear classifier

Passive Learning: training examples selected at random

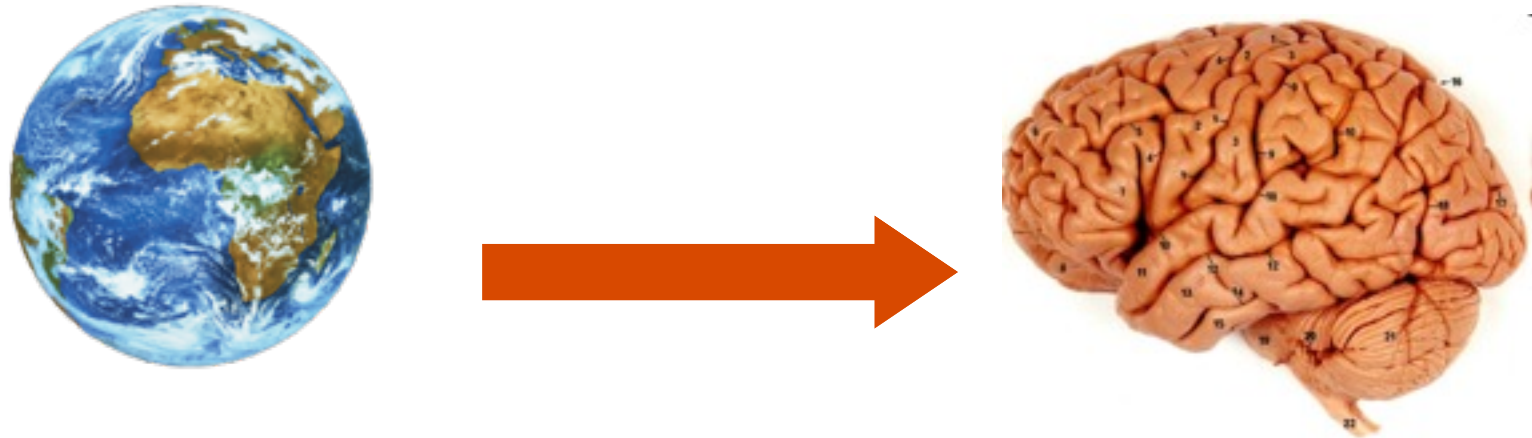
Active Learning: especially informative examples are sequentially selected

Active learning can very effectively "narrow down" the location of the optimal decision boundary

The Theory of the Organism-Environment System: III. Role of Efferent Influences on Receptors in the Formation of Knowledge*

TIMO JARVILEHTO
Department of Behavioral Sciences, University of Oulu, Finland

Abstract—The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, which makes the development of a dynamic living system, the organism-environment system, possible. In this connection process, the efferent influences on receptor activity are of particular significance because, with their help, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of the efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.



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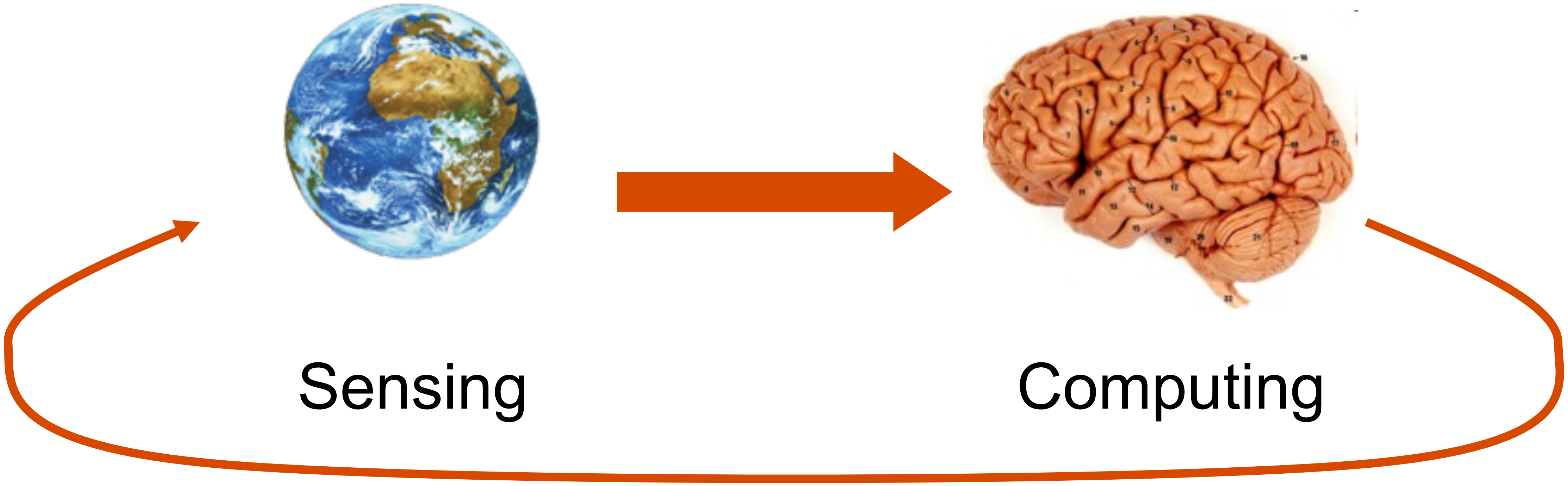

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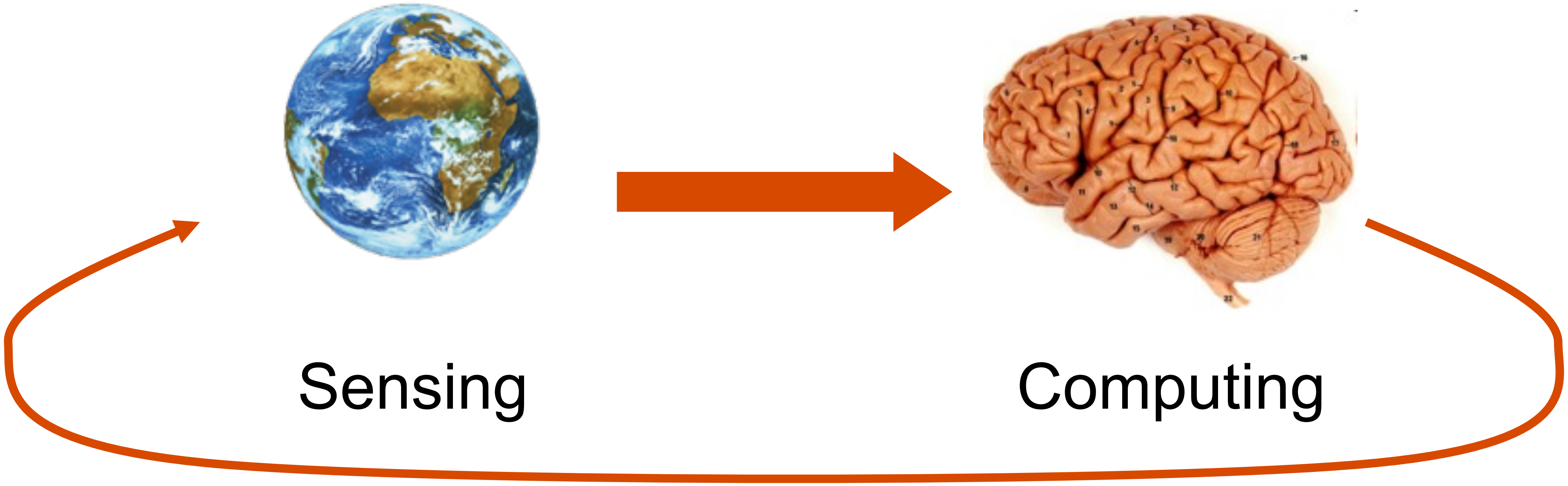

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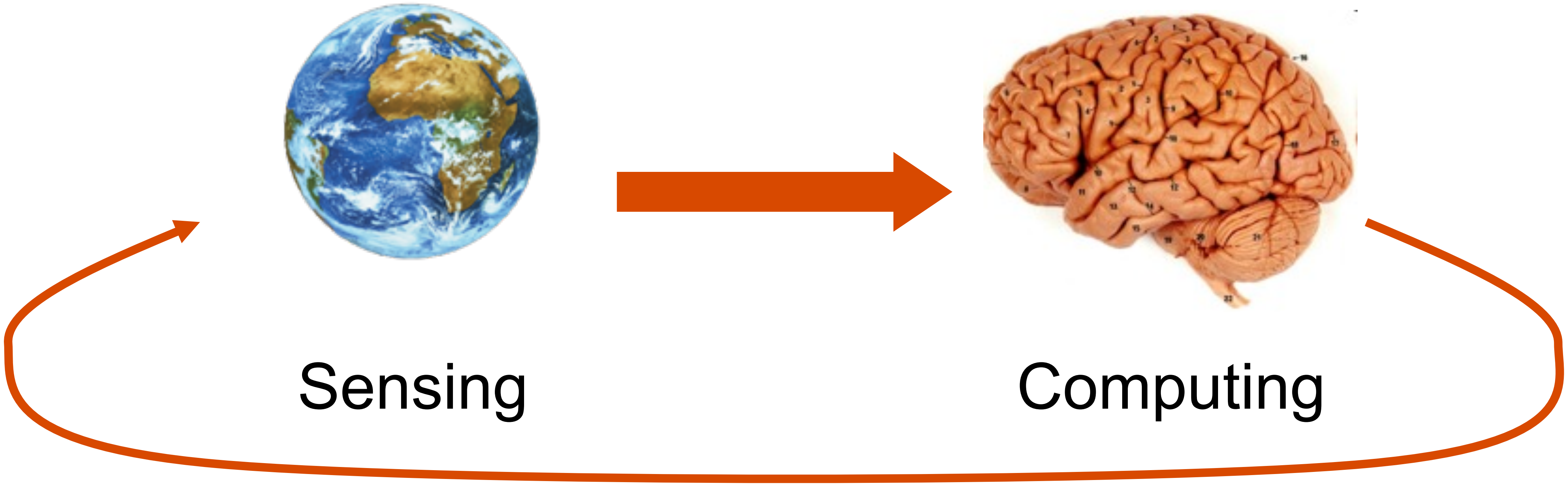

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Visual Perception

Attentional mechanisms probably limit our capacity to about 44 bits per-glimpse (Verghese and Pelli (1992))

So how to we perceive ‘reality’ from so few bits of information?

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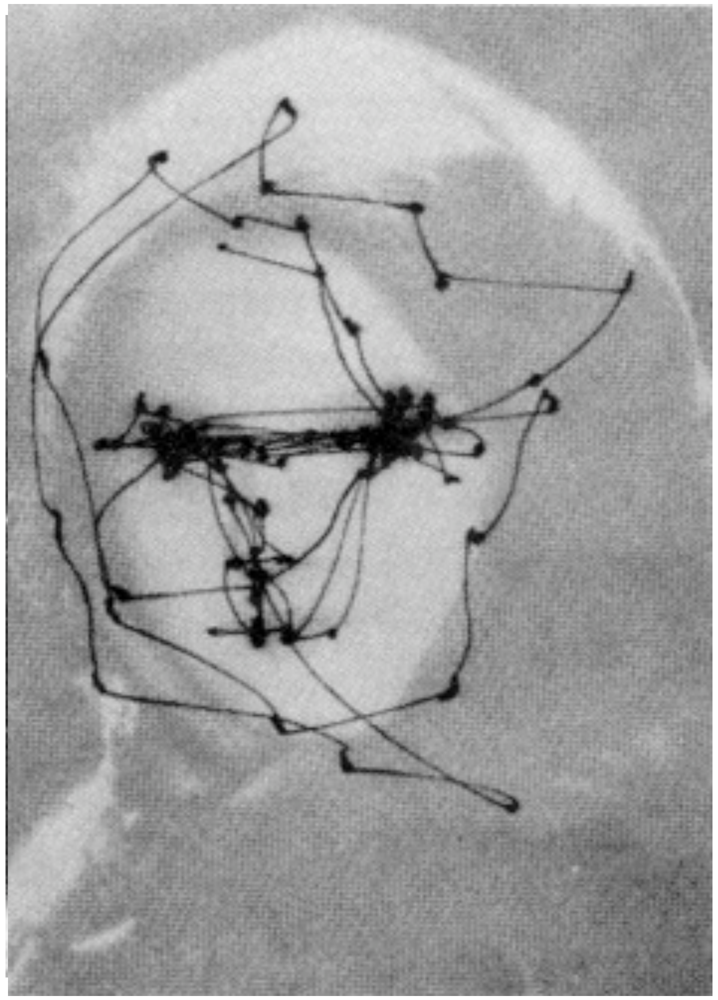
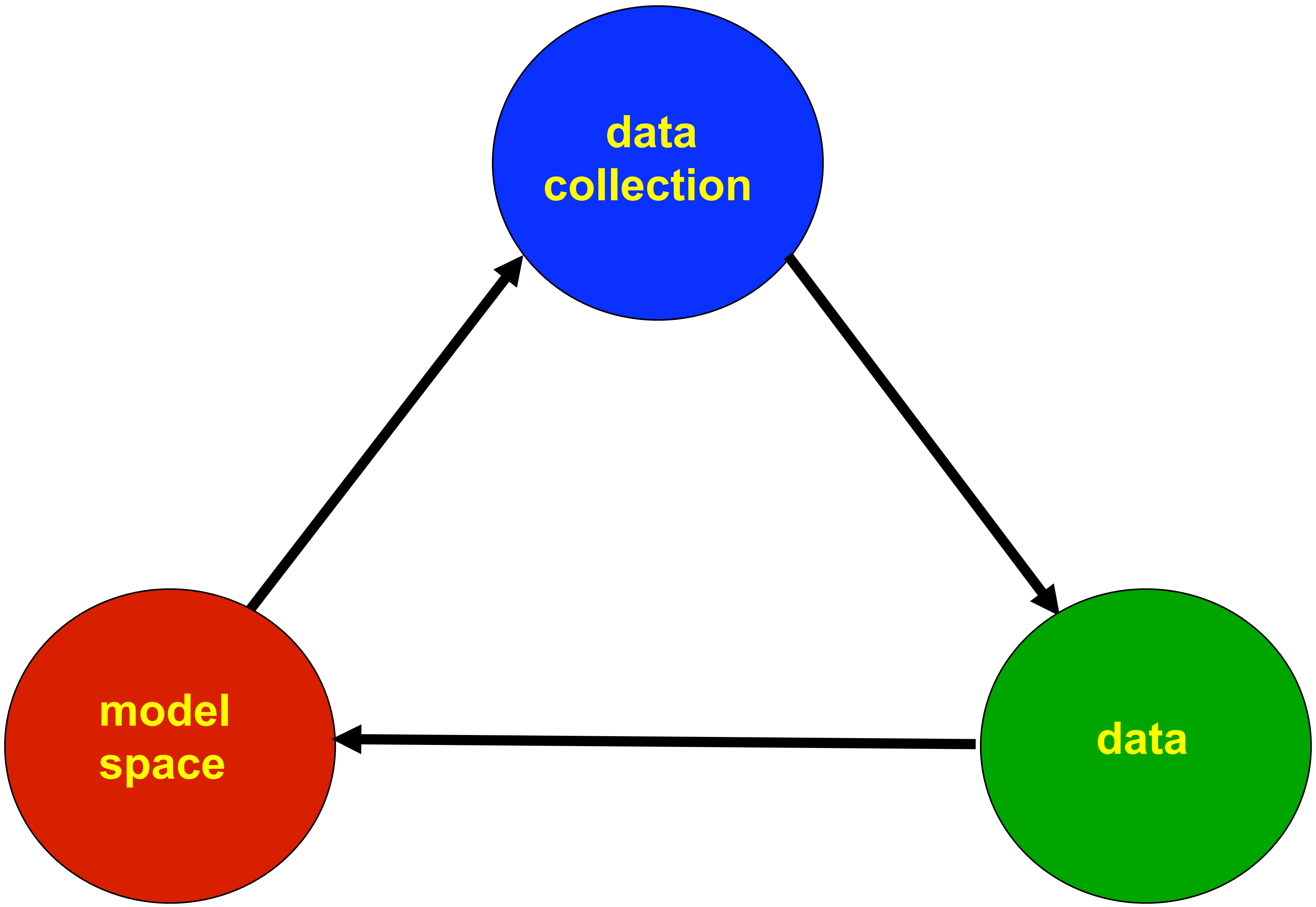



S

Mathematical Theory of Active Sensing and Learning

$\mathcal{Y}$: possible measurements/experiments



$\mathcal{X}$: models/hypotheses
under consideration

$y_1(x), y_2(x), \ldots$: information/data

Adaptive vs. Non-Adaptive: Three Situations

The "bare minimum" number of measurements depends on intrinsic complexity of $\mathcal{X}$. In practice, the minimum number depends on jointly on $\mathcal{X}$ and $\mathcal{Y}$.

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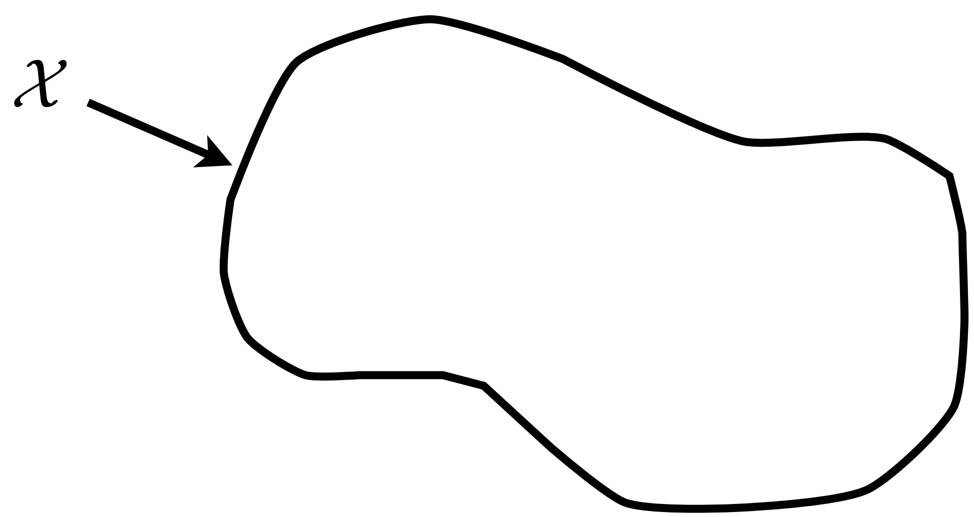
Good and Bad: adaptive requires bare minimum number of measurments, non-adaptive requires many more

The Bare Minimum

Assume $\mathcal{X}$ is equipped with metric d and is compact.



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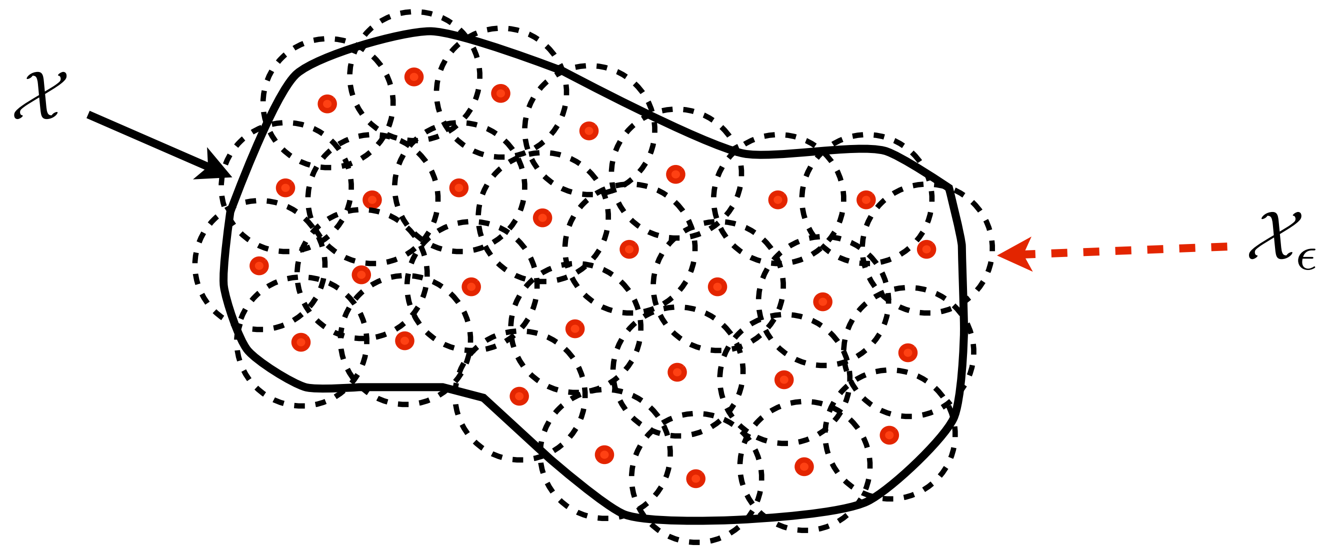

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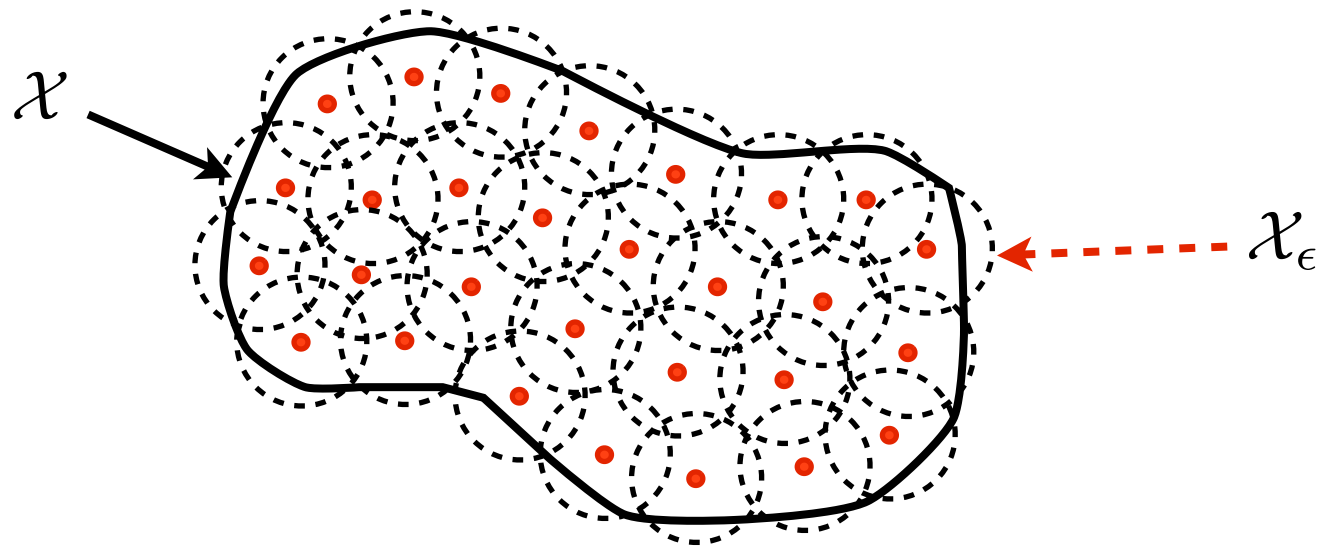

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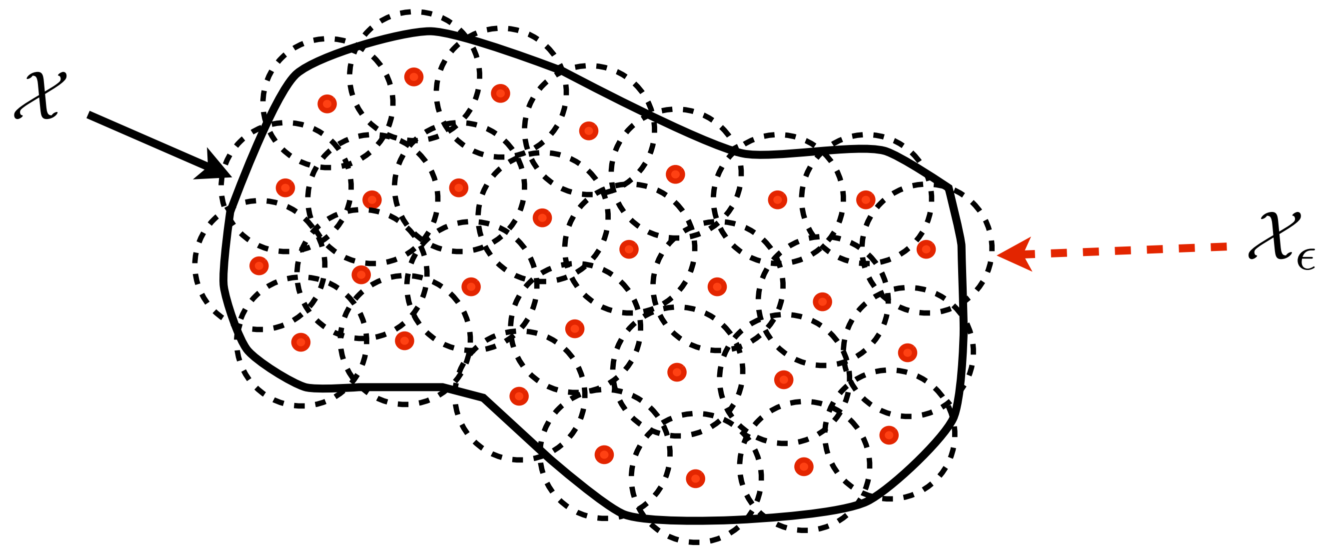

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Ex. suppose $\mathcal{X} = [0,1]^d$. we can take a uniform grid of points spaced ϵ apart as our cover. Then $N_\epsilon = (\frac{1}{\epsilon})^d$ and $\log N_\epsilon = d \log(1/\epsilon)$.

Binary Search

$$\mathcal{X} = \{\text{subsets } [0, \tfrac{1}{N}], [0, \tfrac{2}{N}], \ldots, [0, 1]\}$$
$$\mathcal{Y} = \text{“membership queries”}$$

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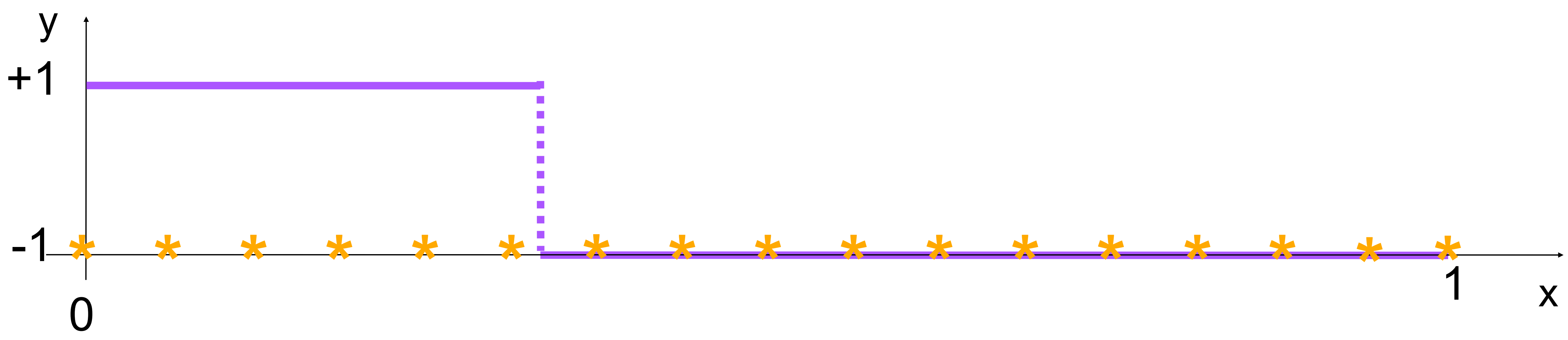

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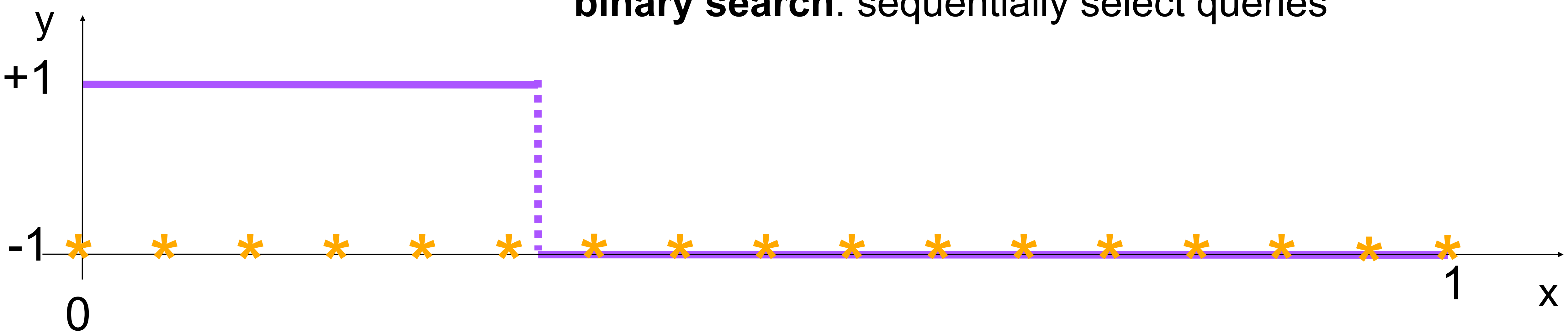

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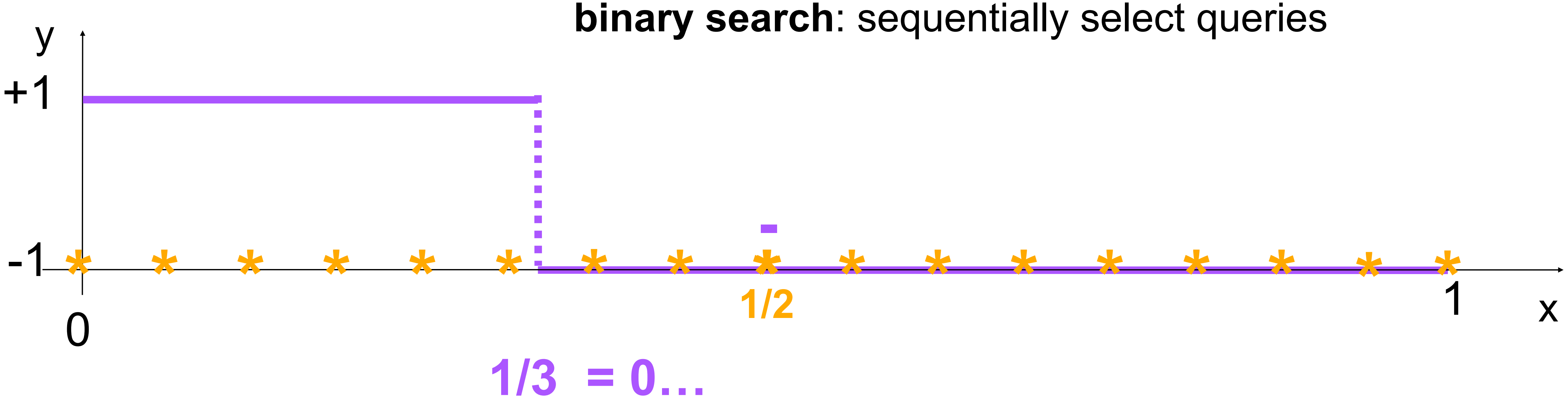

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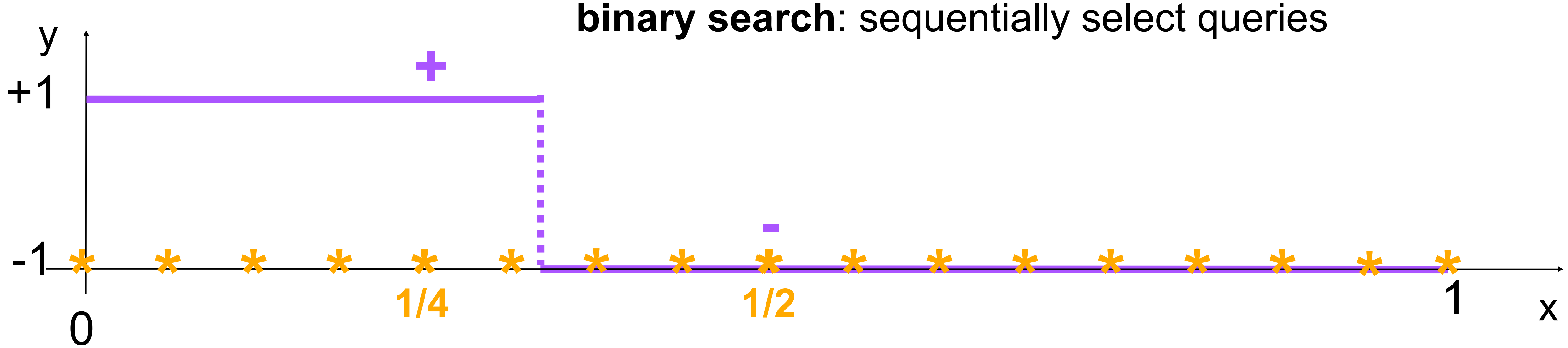

1/3 = 01…

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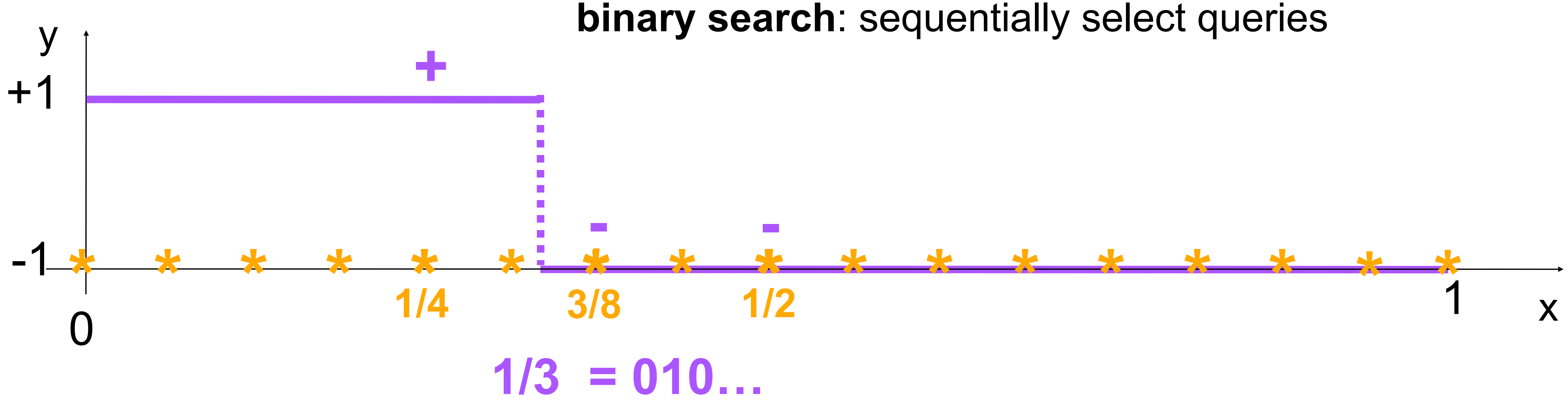

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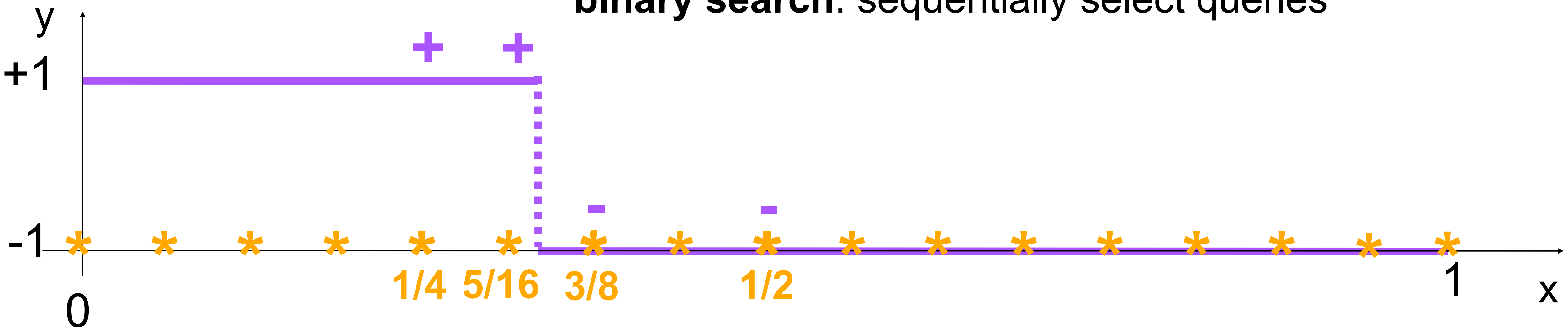

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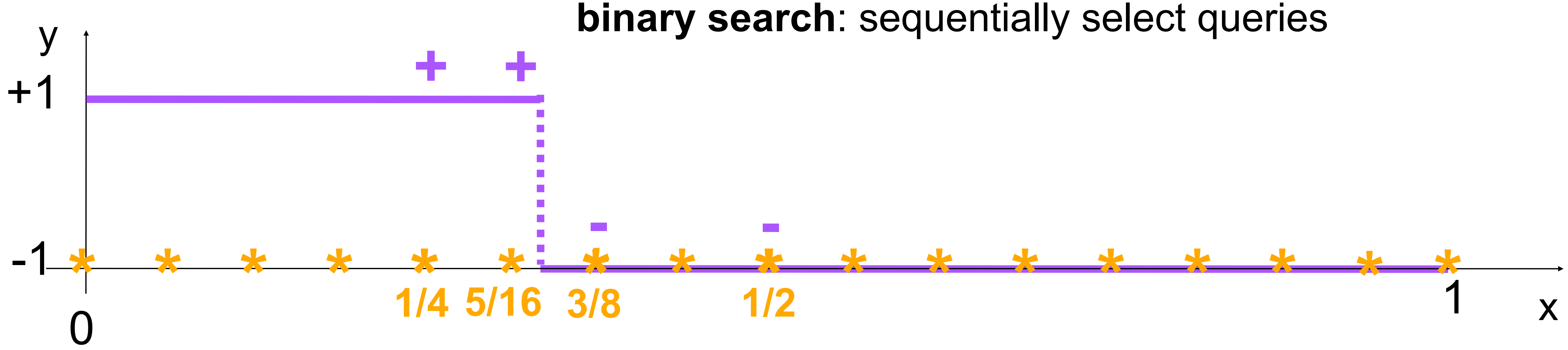

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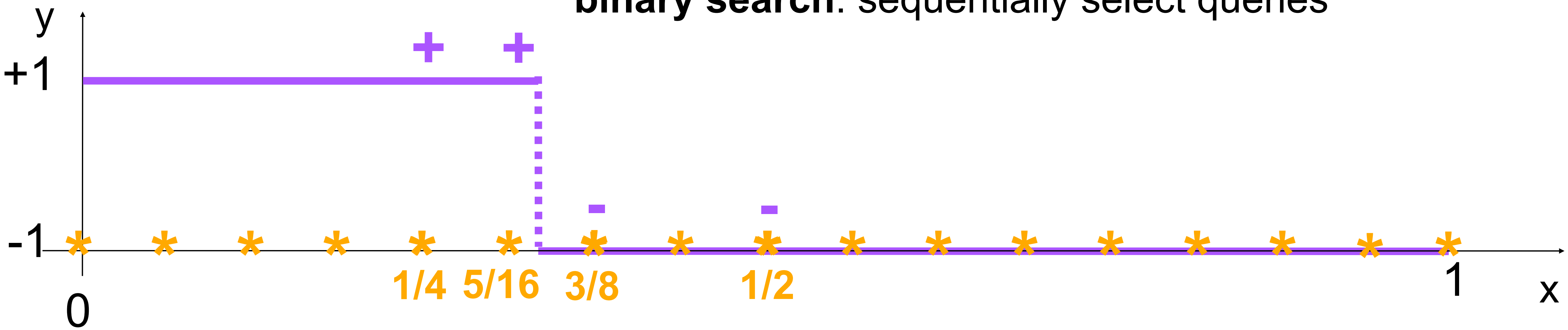

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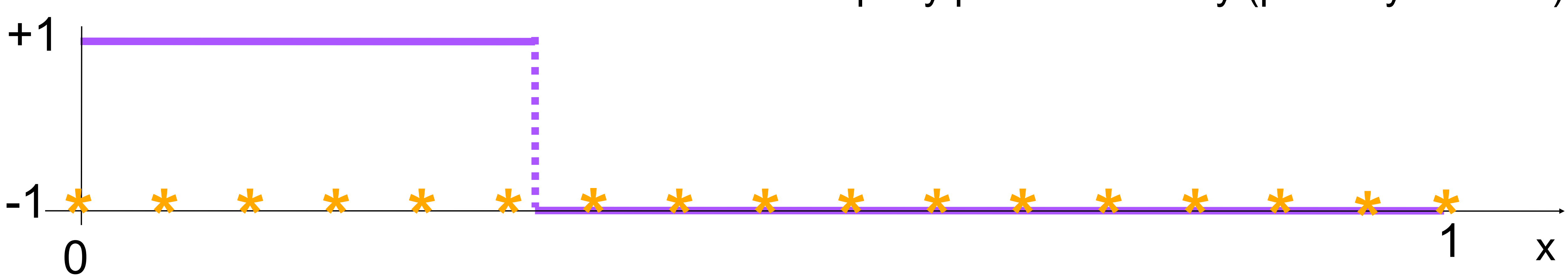
linear search: query points uniformly (possibly random)



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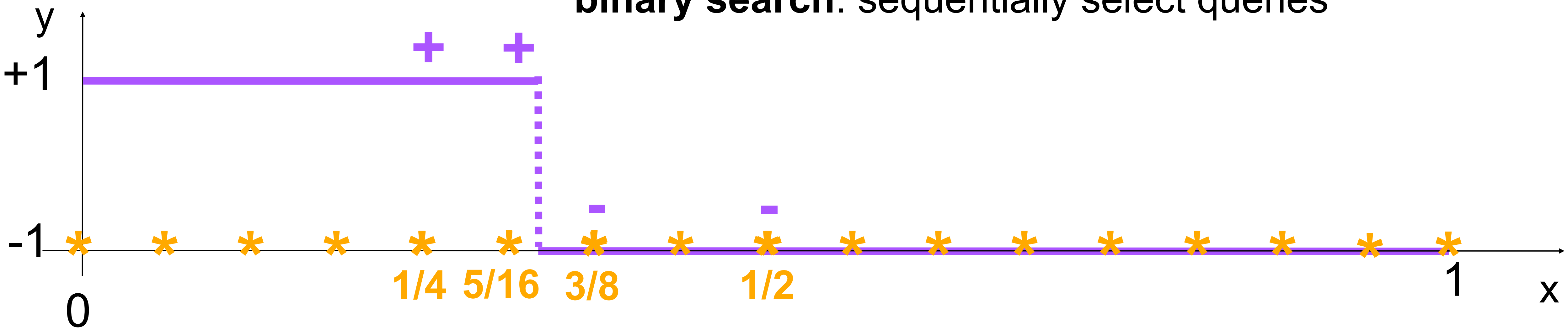

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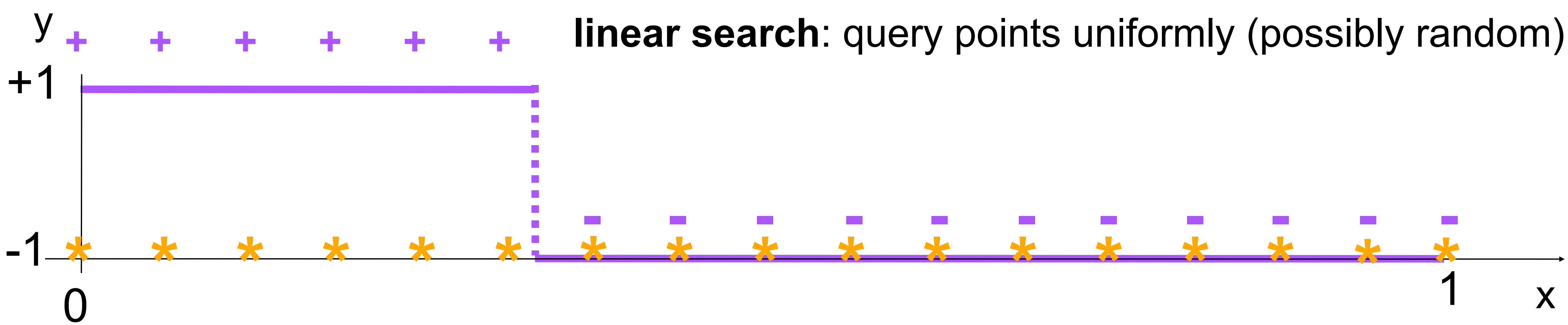

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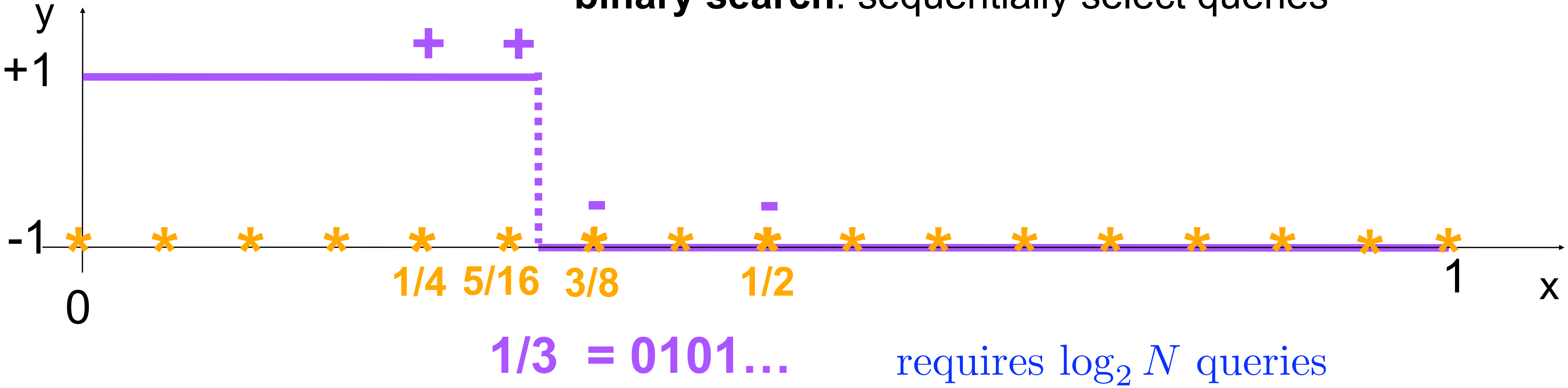

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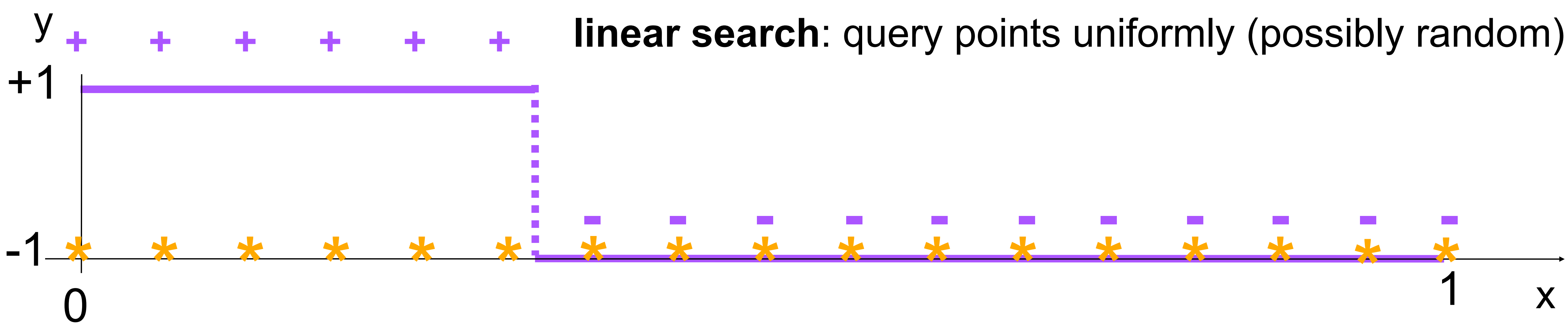

requires $O(N)$ queries

Outline of Tutorial

Part 1: Introduction (Rob), 9:00-9:30

Part 2: Active Sensing (Jarvis), 9:30-10:30

Break, 10:30-10:45

Part 3: Active Learning (Rob), 10:45-11:45

Part 4: Conclusions and Future Directions, 11:45-12