ECE 901
Homework 5

Reconsider the denoising problem consider in **Lecture 15 — Denoising in Smooth Functions Spaces II: Adapting to Unknown Smoothness**. Assume the following observation model:

$$Y_i = f^*(i/n) + W_i, \quad i = 1, \ldots, n$$

where $W_i \overset{i.i.d.}{\sim} N(0, \sigma^2)$ and $f^* : [0,1] \to R$ is Holder-α smooth function for some unknown $\alpha \in (0, 2]$. In class we analyzed a piecewise-linear estimator, on an adaptive uniform partition, which gave a MSE rate of convergence of $O\left(n^{\frac{-2\alpha}{2\alpha+1}} \log n\right)$, automatically adapting to the unknown degree of smoothness α.

A crucial assumption in this example was that the noise variance σ^2 is known. The variance plays an important role in the actual selection of $\widehat{f}_n$ (although it does not affect the rate of convergence).

Suppose that σ^2 is not known. How would you construct $\widehat{f}_n$?

Re-analyze this case in the situation when σ^2 is unknown, and propose a new estimator $\widehat{f}_n$ that automatically adapts to the unknown noise power.