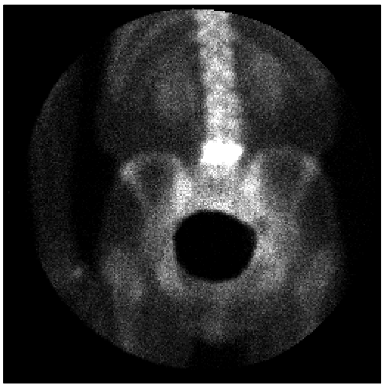
ECE 732
Project 1: Nuclear Medicine Imaging

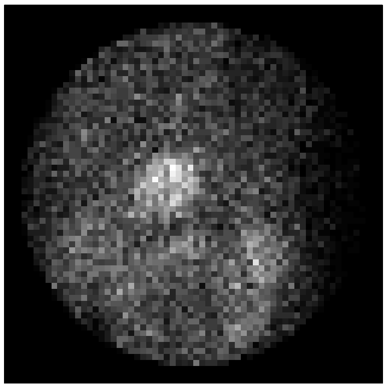
Due: 5pm, October 10, 2003

You can discuss issues and problems in this project with your classmates. However, each student must prepare their own project report, including their individual analysis, Matlab implementations, and experimental results. Do not share your final report or Matlab code with other students. Reports or programs that appear to be copies will receive zero-credit.

This project investigates nuclear medicine imaging. Some of the very basic aspects of nuclear medicine imaging were discussed in the lecture. Two typical nuclear medicine images are shown below, and can be downloaded from the course webpage. The images are very noisy and are blurred to the physical design of the imaging system. The goal of this project is to design image restoration filters to reduce noise and enhance spatial resolution in nuclear medicine.



spine image



heart image

1. The blur is due to the design of the nuclear medicine camera. In nuclear medicine, a radioactive pharmaceutical is injected into the patient. As the radioactive material decays, gamma-rays are emitted from within the patient. The intensity of the emissions (brightness in images) is directly proportional to the uptake of the radioactive pharmaceutical in the patient (e.g., high concentration in areas of bone repair (spine image) or high blood flow (heart image)). The high energy gamma-rays cannot be focused with conventional glass optics, so instead a sort of "pinhole" like imaging system is used. A lead plate with an array of small holes is placed in front of the imaging electronics which absorbs gamma-ray photons that do not pass directly through the holes. This plate is called the *collimator* and it serves as a lens. In effect, only gamma-rays coming from a direction perpendicular to the face of the camera are detected, which allows us to easily determine where the photons were emitted in the patient (just trace the "ray" back from the camera to the patient). However, there is a trade-off in the camera design. Better spatial resolution is achieved by making the collimator very thick with very small holes. On the other hand, we will be able to collect more more photons (and hence achieve a higher SNR) by making the holes larger, but this will cause more blurring (and less spatial resolution).

Practical systems are designed to strike a balance between spatial resolution (blurring) and photon count levels (SNR) through a certain choice for the size of the holes and the thickness of the lead. The collimation effect can be approximated by a convolution. That is, the blurred image is approximately equal to the convolution of the unblurred image with a point spread function (PSF) related to the collimator. The blurring effect is basically like convolving the image with a small, radially symmetric pulse of a certain width. The width and shape of the pulse depends on the hole size and thickness of the collimator, and on the distance of the source (patient) from the face of the camera.

The Fourier transform of a nuclear medicine image $x[m, n]$ is approximately given by

$$X(u, v) \;=\; H(u, v)Y(u, v) + W(u, v),$$

where $H(u, v)$ is the Fourier transform of PSF, and $Y(u, v)$ and $W(u, v)$ are, respectively, the Fourier transforms of the noise-free image we want to recover and the noise due to the random nature of the radioactive decay process. Thus, the simple inverse filter would take the form:

$$\widehat{Y}(u, v) \;=\; \frac{X(u, v)}{H(u, v)}.$$

Derive an expression for the Fourier transform of the PSF, $H(u, v)$, in terms of the collimator specfications and the distance from the source.

2. Construct a Wiener filter for deblurring the nuclear medicine spine and heart images. Assume that the noise is white noise of a certain power σ^2. To specify the Wiener filter you will need to:

- set σ^2 at a certain value
- choose a power spectral density model for the underlying image y
- set the parameters of the PSF (since we don't know them precisely for these images)

How do your settings of the various parameters above affect the quality of the results?

3. The "noise" in nuclear medicine is actually not Gaussian distributed, so the variance is not constant over the entire image. As stated above the randomness in the data is caused by the random nature of the radioactive decay. The data can be well modeled as a realization of a *Poisson* process. Specifically, the image $x[m, n]$ is accurately modeld as a realization from a two-dimensional Poisson process whose intensity function $\lambda[m, n]$ is an image whose Fourier transform is $H(u, v)Y(u, v)$, defined above. One of the key distinguishing features of the Poisson process is that the variance is proportional to the intensity, so in effect the noise level is larger in brighter areas of the image.

The Poisson distribution for the data has an explicit form in terms of the noise-free image $y[m, n]$ and the PSF $h[m, n]$ (inverse Fourier transforms of $Y(u, v)$ and $H(u, v)$, respectively):

$$p(x[m, n]) = \exp\left\{-\sum_{i,j} h[i, j]y[m - i, n - j]\right\} \frac{\left(\sum_{i,j} h[i, j]y[m - i, n - j]\right)^{x[m,n]}}{x[m, n]!}.$$

The value $p(x[m, n])$ is simply the probability that $x[m, n]$ photons will be detected at position m, n assuming that the intensity of photon emissions in the patient is given by y. The joint probability of observing the entire image x is given by $p(x) = \prod_{m,n} p(x[m, n])$. A reasonable way to "estimate" y from x is view $p(x)$ as a function of y (with x, our data, fixed), and then to maximize the function with respect to y. When $p(x)$ is regarded as a function of y it is usually called the *Likelihood function*, and is denoted by $\ell(y)$, rather than $p(x)$. That is,

$$\ell(y) = \prod_{m,n} \exp\left\{-\sum_{i,j} h[i, j]y[m - i, n - j]\right\} \frac{\left(\sum_{i,j} h[i, j]y[m - i, n - j]\right)^{x[m,n]}}{x[m, n]!}.$$

A maximizer of $\ell(y)$ is called a *Maximum Likelihood Estimate*. It simply produces a y that makes the measured image most likely.

Unfortunately, a little thought reveals that finding the maximum of $\ell(y)$ with respect to y is no easy task. There is no closed-form solution. Therefore, one must resort to an iterative method for maximization. For example, one could use *gradient ascent*, the obvious counterpart of gradient descent. In fact, maximizing $\ell(y)$ is the same as minimizing $-\ell(y)$, so you could apply gradient descent to $-\ell(y)$. Another option is the so-called Expectation-Maximization (EM) algorithm. The EM algorithm is especially easy to implement for this problem.

Devise an iterative "restoration" algorithm based on maximizing the Poisson likelihood function with respect to y. Keep in mind that you will need to compute the convolution $\sum_{i,j} h[i,j]y[m-i,n-j]$ often in your iterations, so the FFT can again be of help.

a. Examine the performance as a function of iteration? Does the restoration keep improving as you iterate?

b. Compare the performance of the iterative algorithm to that of the Wiener filter.

References

C. E. Metz, F. B. Atkins, and R. N. Beck, "The geometric transfer function component for scintillation camera collimators with straight parallel holes," *Phys. Med. Biol.* , vol. 25, no. 6, pp. 1059-1070, 1980.

Y. Vardi, L. A. Shepp, L. Kaufman, "A Statistical Model for Positron Emission Tomography," *J. Amer. Stat. Assoc.* , 80 (389), pp. 8-37, 1985.