ECE 732
Homework 8

One way to construct a non-negative window function $W(f)$ is as follows. Define a sequence $\{v(0), v(1), \ldots, v(M-1)\}$ and let $W(f) = |V(f)|^2$.

Now suppose that we wish to choose $v = \{v(k)\}$ so that the sidelobe leakage is minimized. We can formulate this as the following optimization:

$$\max_{v} \frac{\int_{-\beta}^{\beta} W(f)\, df}{\int_{-1/2}^{1/2} W(f)\, df}.$$

The goal of this optimization is the maximize the energy in the main lobe, $f \in [-\beta, \beta]$, relative to the total energy of the window. We can express this ratio in terms of $V(f)$ as

$$\frac{\int_{-\beta}^{\beta} |V(f)|^2\, df}{\int_{-1/2}^{1/2} |V(f)|^2\, df}.$$

Also, notice that $V(f)$ can be expressed as the inner product $v^H a$, where

$$a = [1 \; e^{-j2\pi f} \; \cdots \; e^{-j2\pi(M-1)f}]^T.$$

Using this vector notation and Parseval's theorem, show that the optimal vector v is given by the dominant eigenvector of a certain $M \times M$ matrix.

Based on this result, in Matlab design and plot optimal windows for various values of β and M.