ECE 732
Homework 5

Adaptive Communication Channel Equalization

The bit sequence $b[n]$ is sent by a transmitter to a receiver. Assume that $b[n]$ is a realization of a random sequence taking values $+1$ and -1 with equal probability. Furthermore assume that $b[n]$ is statistically independent of $b[m]$ for all $m \neq n$. Unfortunately the sequence is distorted by a channel on its way to the receiver. The channel is modeled as an LTI system $h[n]$ followed by an additive white noise of power $\sigma^2 = 1$. The z-transform of $h[n]$ has the form $H(z) = \frac{1}{1-\alpha z^{-1}}$, where $|\alpha| < 1$ is an unknown parameter. Let $x[n]$ denote the received sequence.

(a) Derive the Wiener filter that will minimize the MSE at the receiver.

(b) Derive an expression for the gradient descent (GD) algorithm to minimize the MSE. Also, determine the allowable step-size range.

Now suppose that we don't know the noise characteristics, and to identify the channel, a test bit sequence $b[n]$ (known by both the sender and receiver) is sent from the transmitter to the receiver. Assume that the test sequence is a pseudorandom binary sequence.

(c) Derive an expression for the LMS algorithm. What range of step-size would you recommend?

(d) Simulate this problem and implement the GD and LMS algorithms in Matlab. How do the trajectories and learning curves of GD and LMS compare?

(e) Now suppose that α is unknown *and slowly time-varying*. Simulate this situation and evaluate how well the LMS algorithm can track the channel (i.e., plot error vs. time and/or compare the estimated position of the zero in the equalizer with the ideal value).