ECE 732
Advanced Signal Processing
Homework 2

FIR Wiener Filtering

Suppose we observe a signal $s[n]$ with additive white noise process $w[n]$ of variance $\sigma_w^2 = 1$:

$$x[n] = s[n] + w[n].$$

Let us model the signal as a random process generated by passing another independent white noise process $v[n]$ of power σ_v^2 through a single-pole IIR digital filter:

$$s[n] \;=\; \alpha\, s[n-1] \;+\; v[n],$$

where $0 < \alpha < 1$.

(a) Give an expression for the autocorrelation function $R_{ss}[n]$ for the signal $s[n]$ in terms of σ_v^2 and α.

(b) Design a length-2 FIR Wiener filter to estimate $s[n]$ from $x[n]$. That is, design a length-2 filter $h[n]$ so that $\widehat{s}[n] = \sum_{k=0}^{1} h[k]x[n-k]$ is a close to $s[n]$ as possible, in the mean-square error sense. Again, express your solution in terms of α and σ_v^2.

(**continued on next page**)

1. (continued)

(c) Determine the corresponding minimum MSE for this filter (in terms of α and σ_v^2).