ECE 732
Advanced Signal Processing
Homework 1

1. Let $w[n]$ denote a Gaussian white noise sequence. Determine the mean and autocorrelation functions of $x[n]$ in the following cases.

(i) $x[n] = 0.8x[n-1] + 0.5w[n-1]$

(ii) $x[n] = w^3[n]$

(iii) $x[n] = 0.4w[n] - 0.1w[n-1]$

(iv) $x[n] = -0.9x[n-1] + 0.8x[n-2]$

2. Let $x[n]$ be a wide-sense stationary random process and input to a LTI system with known impulse response $h[n]$. Denote the output by $y[n]$. Suppose that the input $x[n]$ is not observable, but that we can measure the output $y[n]$.

(i) Propose an estimate of the power spectrum of $y[n]$ (i.e., describe the processing you would apply to the observed output $y[n]$ to estimate the power spectrum $S_{yy}(\omega)$).

(ii) How would you use the estimate of $S_{yy}(\omega)$ to estimate the power spectrum of the unobservable input $x[n]$?

3. System Identification.

a. Let $x[n]$ be a two-level random process in which each sample is independent of all other samples and has a probability distribution

$$Pr(x[n] = a) = \frac{1}{2} \quad \text{and} \quad Pr(x[n] = b) = \frac{1}{2},$$

where a and b are arbitrary real numbers. Use realizations of this process to "probe" an unknown LTI system. Can you identify the system uniquely from repeated experiments with this type of input process? If so, formulate the procedure you would use.

b. Implement your procedure in MATLAB. Use the `rand` function to generate realizations of the random process above. Apply these realizations to the input of a finite impulse response filter of your choice. Approximate the expectation operator by averaging the results from multiple input-output realizations. Demonstrate that your procedure is capable of identifying the impulse response. You might also want to test your method with a GWN sequence (try the function `randn`).