ECE 532
Homework 5

Due Thursday February 17 at the beginning of class

1. Consider the multivariate Gaussian binary classification problem in which $\mu_1 = \mu_0$, but $\Sigma_1 \neq \Sigma_0$.

- **a.** Derive a expression for the discriminant functions in this case.
- **b.** Sketch the form(s) in the two-dimensional setting (draw contour plots of class-conditional densities and resulting decision boundaries).

2. Consider the bivariate (2-dimensional) Gaussian binary classification problem in which $\Sigma_1 = \Sigma_0$, but $\mu_1 \neq \mu_0$. Write a Matlab script that performs the following operations:

- **a.** Given a covariance matrix Σ, mean vectors μ_1 and μ_0, and prior probabilities $p_0 = 1 - p_1$, determine the minimum probability of error decision boundary in the plane.
- **b.** Generate $p_i \times 100$ i.i.d. realizations ($i = 0, 1$) from each class-conditional density and display the scatter plot of these points with the decision boundary superimposed.

Demonstrate your program by generating plots for at least two different covariance matrices. In each case, also consider at least two different settings for the prior probabilities.

3. Consider the following frequency-shift-keying (FSK) binary communication system. We send the binary digit 0 by transmitting the sequence

$$s_0(n) = \cos\left(2\pi \frac{m_0}{N} n\right), \quad n = 0, 1, \ldots, N-1$$

and send the binary digit 1 by transmitting

$$s_1(n) = \cos\left(2\pi \frac{m_1}{N} n\right), \quad n = 0, 1, \ldots, N-1$$

Assume that $m_0 \neq m_1$ are both integers. The received signal is $x(n) = s_y(n) + w(n)$, where $y = 0$ or $y = 1$ and $w(n)$ is a sequence of iid samples from a $\mathcal{N}(0, \sigma^2)$ distribution. Let $x = [x(0), \ldots, x(N-1)]^T$.

- **a.** Derive an optimum classifier $f(x)$ for deciding if a 1 or 0 was transmitted (at this point, the threshold is arbitrary since we haven't defined which "optimality" we desire).
- **b.** Let $N = 32$, $m_0 = 0$, and $m_1 = N/2$. Plot the ROC curves of $P(f(X) = 1|Y = 1)$ vs. $P(f(X) = 1|Y = 0)$ for $\sigma^2 = 0.01, 0.1, 1$, and 10.
- **c.** Derive the optimal classifier if instead of white noise, the noise samples were correlated with covariance matrix $\mathbf{R}$.
- **d.** Describe how this signaling scheme could be generalized for transmitting M-ary data consisting of M symbols.