ECE 532
Homework 2

Due Thursday February 3 at the beginning of class

1. Consider a two-dimensional random vector $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ where X_1 is uniformly distributed over the interval $[0, 1]$ (i.e., $p(x_1) = I_{[0,1]}(x_1)$) and X_2 is uniformly distributed over the interval $[3/8, 5/8]$ (i.e., $p(x_2) = 4I_{[3/8,5/8]}(x_2)$). Furthermore, assume that X_1 and X_2 are statistically independent. Note: The indicator function $I_A(x)$ takes the value 1 if x belongs to the set A and zero otherwise.

a. Sketch the joint density $p(x_1, x_2)$.

b. Use the Matlab `rand` command to generate 100 realizations of the random variables X_1 and X_2. Visualize the distribution in Matlab by plotting the pairs (x_1, x_2) in the unit square.

c. Compute the expectations $E[X_1]$, $E[X_2]$, and $E[X_1 X_2]$.

2. Consider a two-dimensional random vector where X_1 is uniformly distributed over the interval $[0, 1]$ (i.e., $p(x_1) = I_{[0,1]}(x)$) and the conditional density for X_2 given X_1 is

$$p(x_2|x_1) = \frac{2x_2}{x_1^2} I_{[0,x_1]}(x_2)$$

Note: The indicator function $I_A(x)$ takes the value 1 if x belongs to the set A and zero otherwise.

a. Derive an expression for $p(x_2)$, the density of X_2.

b. Compute the expectations $E[X_1]$, $E[X_2]$, and $E[X_1 X_2]$.

3. Consider a two-dimensional jointly Gaussian random vector $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, with mean vector $\mu = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$ and covariance matrix $\Sigma = \frac{1}{1000} \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$.

a. Give an expression for $p(x_2)$, the density of X_2.

b. Let $Y = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ be jointly Gaussian with mean vector $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and covariance matrix $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Determine a linear transformation of the form $AY + b$ so that the resulting random vectors are distributed like X. You may use the Matlab function `eig` to determine the proper transformation.

c. Using the linear transformation determined above, use the Matlab command `randn` to generate 100 pairs of independent Gaussian distributed variables with mean zero and variance 1. Apply the transformation to obtain pairs distributed like X. Visualize the distribution in Matlab by plotting the resulting pairs (x_1, x_2) in the unit square.