ECE 431 Homework 8
Due: Friday, November 10, 2006

Problem 4.57, Oppenheim, Schafer and Buck: As discussed in lecture, to process sequences on a digital computer, we must quantize the amplitude of the sequence to a set of discrete levels. This quantization can be expressed in terms of passing the input sequence $x[n]$ through a uniform quantizer $Q(x)$. As discussed in the notes, if the quantization interval Δ is small compared with the changes in the level of the input sequence, we can assume that the output of the quantizer is of the form

$$y[n] = x[n] + e[n]$$

where $e[n] = Q\left(x[n]\right) - x[n]$. Assume that $x[n]$ is a stationary white-noise process with zero mean and variance σ_x^2, and model $e[n]$ as a stationary random process with a probability density uniformly distributed between $-\Delta/2$ and $\Delta/2$, uncorrelated from sample to sample and independent of $x[n]$.

a. Find the mean, variance, and autocorrelation of $e[n]$.

b. What is the signal-to-quantization-noise ratio σ_x^2/σ_e^2?

c. The quantized signal $y[n]$ is to be filtered by a digital filter with impulse response $h[n] = \frac{1}{2}\left[a^n + (-a)^n\right] u[n]$. Determine the variance of the noise produced at the output due to the quantization noise, and determine the signal-to-noise ratio at the output.

In some cases we may want to use nonlinear quantization steps, for example, logarithmically spaced steps. This can be accomplished by applying uniform quantization to the logarithm of the input as depicted in the figure below, where $Q[\cdot]$ is the same uniform quantizer that we considered above. In this case, if we assume that Δ is small compared with the changes in the sequence $\ln(x[n])$, then we can assume that the output of the quantizer is

$$\ln(y[n]) = \ln(x[n]) + e[n] \quad \text{or equivalently} \quad y[n] = x[n]\exp([e[n]).$$

For small errors, we can approximate $\exp(e[n])$ by $(1 + e[n])$, the first-order Taylor series approximation of the exponential function. This approximation results in

$$y[n] \approx x[n](1 + e[n]) = x[n] + f[n].$$

This equation will be used to describe the effect of logarithmic quantization. Assume $x[n]$ and $e[n]$ are random signals with the same distributions as described above.

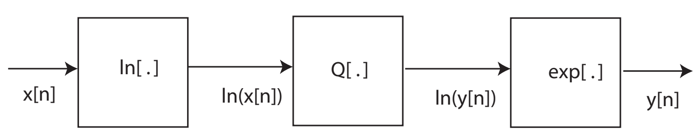

Figure P4.57-1

d. Determine the mean, variance and autocorrelation of the *additive* noise $f[n] = x[n]e[n]$.

e. What is the signal-to-quantization-noise ratio σ_x^2/σ_f^2? Note that in this case σ_x^2/σ_f^2 is idenpendent of σ_x^2. Therefore, within the limits of our assumptions, the signal-to-quantization-noise ratio is independent of the input signal level, whereas for linear quantization, the ratio σ_x^2/σ_e^2 depends directly on σ_x^2.

f. The quantized signal $y[n]$ is to be filtered by a digital filter with impulse response $h[n] = \frac{1}{2}\left[a^n + (-a)^n\right] u[n]$. Determine the variance of the noise produced at the output due to the quantization noise, and determine the signal-to-noise ratio at the output.