ECE 431
Digital Signal Processing
Homework 7

Due Friday, November 3, 2006

1. Let $w[n]$ denote a Gaussian white noise sequence with mean zero and variance 1. Determine the mean and autocorrelation functions of $x[n]$ in the following cases. **Hints:** Note that all the cases below are time-invariant definitions (i.e., $x[n]$ is generated by the same fixed rule at all times), therefore the random signal $x[n]$ will be stationary. In each case, use the definition of the autocorrelation, $R[m] = E[x[n]x[n-m]]$, the definition of $x[n]$ in terms of its past samples and/or the noise, and exploit the fact that the noise is independent. For example, in case (i) $E[x[n]] = 0.8E[x[n-1]] + E[w[n]] = m_x = 0.8m_x$ which implies that $m_x = 0$. Also in case (i), $E[x[n-1]w[n]] = E[x[n-1]] \times E[w[n]] = 0$, since $x[n-1]$ is a function past inputs and past noise values, and is therefore independent of $w[n]$.

(i) $x[n] = 0.8x[n-1] + 0.5w[n]$

(ii) $x[n] = w^3[n]$

(iii) $x[n] = 0.4w[n] - 0.1w[n-1]$

2. Let $x[n]$ be a wide-sense stationary random process and input to a LTI system with known impulse response $h[n]$. Denote the output by $y[n]$. Suppose that the input $x[n]$ is not observable, but that we can measure the output $y[n]$.

(i) Propose an estimate of the power spectrum of $y[n]$ (i.e., describe the processing you would apply to the observed output $y[n]$ to estimate the power spectrum $S_{yy}(\omega)$). **Hint:** Consider using the DFT to estimate the "power" of the output at each frequency.

(ii) How would you use the estimate of $S_{yy}(\omega)$ to estimate the power spectrum of the unobservable input $x[n]$?

3. System Identification.

a. Let $x[n]$ be a two-level random process in which each sample is independent of all other samples and has a probability distribution

$$Pr(x[n] = 1) \;=\; \frac{1}{2} \qquad \text{and} \qquad Pr(x[n] = -1) \;=\; \frac{1}{2},$$

Use realizations of this process to "probe" an unknown LTI system. Can you identify the system uniquely from repeated experiments with this type of input process? If so, formulate the procedure you would use.

Hint: Instead of working with the autocorrelation function $R_{yy}[m] = E\left[y[n]y[n+m]\right]$ of the output random signal $y[n]$, analyze the properties of the cross-correlation function between the input $x[n]$ and the output, that is $R_{xy} = E\left[x[n]y[n+m]\right]$.

b. Implement your procedure in MATLAB. Use the `rand` function to generate realizations of the random process above. Apply these realizations to the input of a filter with finite impulse response ($h[0]$, $h[1]$, $h[2]$, $h[3]$) = (1, 0.5, 0.25, 0.125). Approximate the expectation operator by averaging the results from multiple input-output realizations. Demonstrate that your procedure is capable of identifying the impulse response. In addition, test your method using GWN sequences as the input (generated using the function `randn`).