ELEC 431
Digital Signal Processing
Homework 6

Due Friday, October 27, 2006

1. By hand, calculate the 2d convolution of the image

$$\mathbf{f} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the filter (point spread function)

$$\mathbf{h} = \begin{bmatrix} -1 & -1 \\ +1 & +1 \end{bmatrix}$$

What does toperation does this filter accomplish? Also calculate the 2d *circular* convolution of the image with the filter.

2. Use Matlab's built-in `conv2` function to compute the convolutions between the image `camera.mat`, which can be downloaded from the course website, and each of the PSFs below. Plot, discuss, and interpret the action of each.

$$\mathbf{h}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} /4$$

$$\mathbf{h}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} /25$$

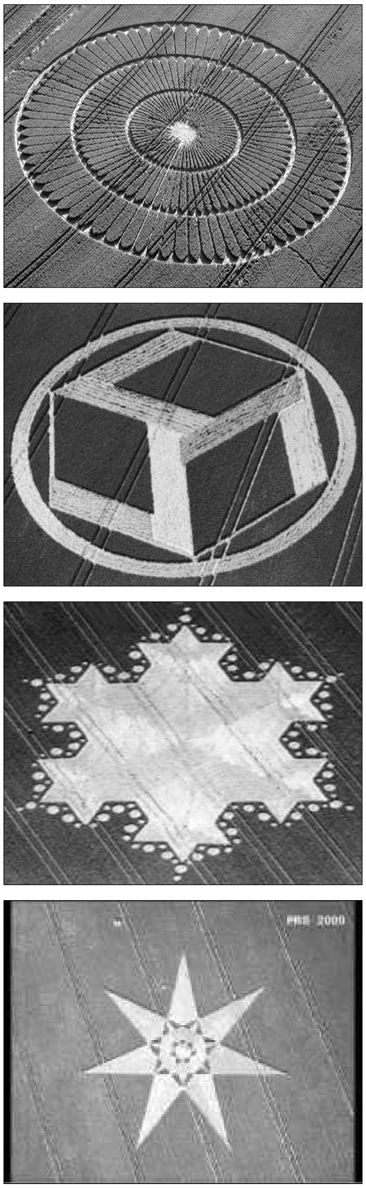
$$\mathbf{h}_3 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} /16$$

How is $\mathbf{h}_3$ related to $\mathbf{h}_1$?

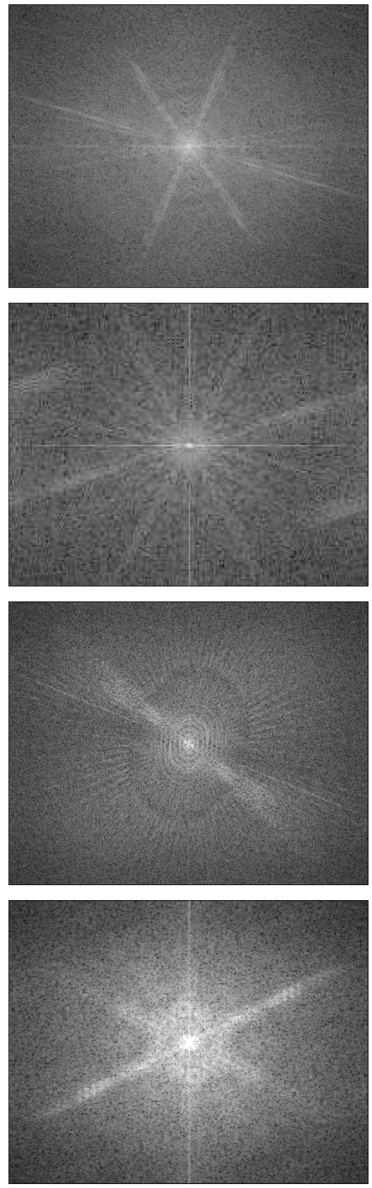
$$\mathbf{h}_4 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \qquad \mathbf{h}_5 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{h}_6 = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

3. This problem investigates crop circles and their 2D Fourier transforms. Four crop circle images are depicted below on the right, along with their four Fourier transforms on the left. The images and transforms are in a random order. Decide Fourier transform matches with which image and explain your answer.



images



Fourier transforms (log-magnitude, DC at center)

4. This problem investigates image deblurring. An ideal sattelite image is depicted below (This is an image of Nimes, France). A blurred version is shown next to it. The blurring could be caused by a number of factors including atmospheric distortions. Typically the blur is not this significant, but in adverse conditions it can be quite severe. Both images can be downloaded from course website. The file containing the ideal image is called `nimes.mat` and the blurred image is in `nimes_b.mat`.

ideal image

blurred image

The point spread function (PSF) $h[m,n]$ of the blurring operator is known in this case, and is contained in the file `blur.mat`. The inverse PSF can be derived from $h[m,n]$. The inverse PSF is in the file `blurinv.mat`. Both PSF files can also be downloaded from the course website.

a. Verify that the inverse PSF is indeed the inverse of the blurring PSF $h[m,n]$ by convolving the two together (in Matlab).

b. Using the Matlab `conv2` function, apply the inverse PSF to the blurred image. Use the Matlab commands `tic` and `toc` to determine the run-time of this convolution operation.

c. Apply the inverse PSF to the blurred image in the DFT domain using the 2D FFT (Matlab command `fft2`). Compare the run-time in this case with the speed of `conv2` determined above.

d. In order to obtain the regular convolution from the FFT-based procedure, we must zero pad the original image. What amount of zero-padding is required? How significant is the "wrap-around" effect of circular convolution if zero-padding is not performed?

e. Another blurred version of the Nimes image is contained in the file `nimes_bn.mat`. A small amount of noise was added to this image, which could be due to instrumentation noise, atmospheric effects, and quantization. The noise is imperceptible to the eye, but has a dramatic effect on the deblurring process. Apply your FFT-based deblurring process to this image. Comment on the result. Can you come up with an improved deblurring process that deals the noise more effectively?