ECE 431
Digital Signal Processing
Homework 4

Due Friday, October 6, 2006

1. Compute, by hand, the circular convolution of the 4-point signals $x[n] = \cos(n\pi/4)$ and $h[n] = (1/2)^n$, $n = 0, \ldots, 3$. Verify your result by computing the DFTs of $x[n]$ and $h[n]$ in Matlab using the command `fft`, multiplying the DFTs, and taking the inverse DFT using the command `ifft`. Include your Matlab code and a Matlab "stem" plot of the circular convolution result.

2. Suppose that we multiply two N-point signals in time. Show that the DFT of the resulting N-point signal is proportional to the circular convolution of the DFTs of the original two signals.

3. **MEGABASS**: Download the rap song from www.ece.wisc.edu/~nowak/ece431/hw/`rap.mat` The signal is 1048576 samples in length. The sampling rate was 44.1kHz. You can play the clip by executing the commands `load rap` followed by `sound(y,Fs,bits)` in Matlab.

a. Given the length and the sample rate, how many seconds of the song were recorded?

b. Use the Matlab command `fft` to compute the DFT of the signal and plot its spectrum (plot the log of the magnitude of the DFT). Do you observe a distinct drop in energy above a certain frequency? What would you suggest is the "effective" bandwidth of the signal? Was it oversampled?

c. To "boost" the bass, amplify the low frequency components of the signal corresponding to frequencies at 500Hz and below by a factor of 3. Determine which DFT coefficients should be amplified given this specification. Listen to the boosted track and plot its spectrum to compare with the original spectrum plotted in b.

d. If we constructed a DSP Megabass system that processed 1048576 length "blocks" of the full-length rap song one at a time and boosted the bass using the DFT method above, then what would be the time delay through the overall system?

4. **Time and Band Limited Signals**: It is a fact that no timelimited signal is bandlimited, and no bandlimited signal can be timelimited. This problem explores this matter, which is an instance of the Heisenberg Uncertainty Principle.

   a. Consider the timelimited "window" function defined to be

$$w_T(t) = \begin{cases} \frac{1}{T}, & |t| \leq T/2 \\ 0, & |t| > T/2 \end{cases}$$

   where $T > 0$ is a given time duration. Compute the CTFT of $w_T(t)$ and discuss its extent.

   b. Now suppose that we observe an arbitrary signal $x(t)$, but only over the finite period $|t| \leq T/2$. Express the CTFT of the "windowed" signal

$$x_T(t) = \begin{cases} \frac{1}{T}x(t), & |t| \leq T/2 \\ 0, & |t| > T/2 \end{cases}$$

   in terms of the CTFT of the CTFTs of the original signal $x(t)$ and the window function $w_T(t)$. Use this result to prove that no timelimited signal can be bandlimited except for the signal $x(t) = 0$.

   c. Consider the fact that every verbal utterance you make must begin and end. There is a time at which you begin to speak and later you stop. Based on the analysis above, we can conclude that your speech is *not* bandlimited. Therefore, your speech must include sinosoidal waves with infinitely high frequencies. Is this reasonable? If our ears could hear sounds at arbitrarily high frequencies, would we hear them? Explain your answer.

   d. Assume that a signal $x(t)$ is bandlimited to the frequency band $|\Omega| \leq B$, but for obvious practical reasons we can only measure it over a finite duration, say $|t| < T/2$. Given that the signal must have infinite extent in time, you might be tempted to conclude that we will "miss" most of the signal. However this isn't necessarily the case. Consider again the CTFT of the window function $w_T(t)$ and denote it by $W_T(\Omega)$. Show that magnitude of $W_T(\Omega)$ can be bounded by the function $2/(\Omega T)$. Argue that for a large enough value of T, the magnitude of the CTFT of $x_T(t)$ outside the frequency band $|\Omega| \leq 2B$ is close to zero. Use this to conclude that timelimted signals (of long enough duration) are *essentially* bandlimited for all practical purposes.