1. Digital Differentiator:

The goal of this problem is to design a DT filter that mimics the CT differentiation operator. Given an input \( x(t) \) the desired output is \( \frac{dx(t)}{dt} \).

**a.** Recall that the differentiation operator is a LTI system. Let \( G_c(\Omega) \) denote the frequency response (CTFT) of the differentiation operator. Derive an expression for \( G(\Omega) \). (HINT: Compare the CTFTs of \( x(t) \) and \( \frac{dx(t)}{dt} \).)

**b.** Recall that the definition of the derivative is

\[
\frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t) - x(t - h)}{h}
\]

This suggests the DT approximation

\[
\frac{dx(t)}{dt} \approx \frac{\Delta x[n]}{T} = \frac{x[n] - x[n - 1]}{T} = \frac{x(nT) - x((n - 1)T)}{T}
\]

What is the frequency response of the DT filter \( g[n] = \delta[n] - \delta[n - 1] \)? Plot the magnitude and phase response of the filter in Matlab.

**c.** How does magnitude and phase of \( G(\omega) \) compare to that of \( G_c(\Omega) \)?

**d.** Consider another DT approximation to the differentiation operator:

\[
h[n] = \frac{\delta[n + 1] - \delta[n - 1]}{2T}
\]

Compute the frequency response of this filter, plot the magnitude and phase, and compare these characteristics to the desired response \( G_c(\Omega) \). This filter’s phase response should match the desired phase much better than that of \( g[n] \).

**e.** Suppose \( x(t) \) is bandlimited to ±1 kHz. How fast should we sample in order to guarantee that \( h[n] \) reasonably approximates a CT differentiator?

2. Consider the DSP system depicted below.

![DSP System Diagram]

The desired filtering is the bandpass filter shown below. The horizontal axis units are Hz (that is, we want a bandpass filter to pass frequencies \((-6000, -5000) \) and \((5000, 6000) \) Hz). Assume the input \( x(t) \) is bandlimited to ±10kHz, and that ideal sampling and reconstruction are performed by the A/D and D/A with sampling period \( T = 5 \times 10^{-5} \) samples/second.
a. Sketch $G(\omega)$ that will produce the desired filtering.

b. Suppose that $x[n]$ is $N$ samples in duration and that we are going to implement $G(\omega)$ in the DFT domain. Which DFT coefficients will you keep and which will you set to zero?