ECE 431
Digital Signal Processing
Homework 3

Due Friday, September 29, at beginning of class

1. Digital Differentiator:

The goal of this problem is to design a DT filter that mimics the CT differentiation operator. Given an input $x(t)$ the desired output is $\frac{dx(t)}{dt}$.

a. Recall that the differentiation operator is a LTI system. Let $G_c(\Omega)$ denote the frequency response (CTFT) of the differentiation operator. Dervive an expression for $G(\Omega)$. (HINT: Compare the CTFTs of $x(t)$ and $\frac{dx(t)}{dt}$.)

b. Recall that the definition of the derivative is

$$\frac{dx(t)}{dt} = \lim_{h \to \infty} \frac{x(t) - x(t-h)}{h}$$

This suggests the DT approximation

$$\frac{dx(t)}{dt} \approx \frac{\Delta x[n]}{T} \equiv \frac{x[n] - x[n-1]}{T} = \frac{x(nT) - x((n-1)T)}{T}$$

What is the frequency response of the DT filter $g[n] = \frac{\delta[n]-\delta[n-1]}{T}$? Plot the magnitude and phase response of the filter in Matlab.

c. How does magnitude and phase of $G(\omega)$ compare to that of $G_c(\Omega)$?

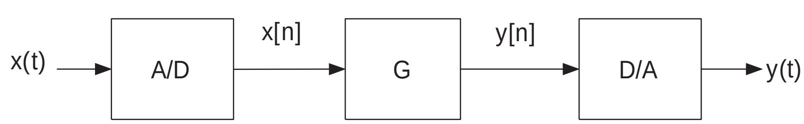
d. Consider another DT approximation to the differentiation operator:

$$h[n] = \frac{\delta[n+1] - \delta[n-1]}{2T}$$

Compute the frequency response of this filter, plot the magnitude an phase, and compare these characteristics to the desired response $G_c(\Omega)$. This filter's phase response should match the desired phase much better than that of $g[n]$.

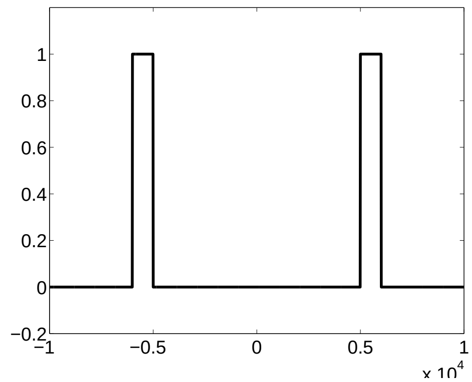
e. Suppose $x(t)$ is bandlimited to ± 1 kHz. How fast should we sample in order to guarantee that $h[n]$ reasonably approximates a CT differentiator?

2. Consider the DSP system depicted below.



The desired filtering is the bandpass filter shown below. The horizontal axis units are Hz (that is, we want a bandpass filter to pass frequencies $(-6000, -5000)$ and $(5000, 6000)$ Hz).

Assume the input $x(t)$ is bandlimited to ± 10kHz, and that ideal sampling and reconstruction are performed by the A/D and D/A with sampling period $T = 5 \times 10^{-5}$ samples/second.



a. Sketch $G(\omega)$ that will produce the desired filtering.

b. Suppose that $x[n]$ is N samples in duration and that we are going to implement $G(\omega)$ in the DFT domain. Which DFT coefficients will you keep and which will you set to zero?