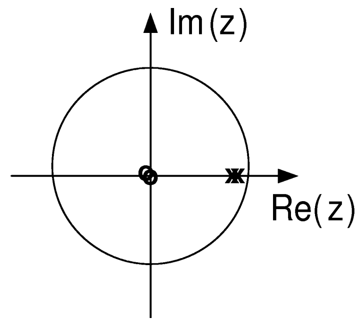
ELEC 431
Digital Signal Processing
Homework 12

In-Class Example, December 15, 2006

In this problem you will investigate the design and application of the basic Wiener filter.

a. Suppose that a "smooth" (low frequency dominant) signal $s[n]$ is generated by passing an Gaussian white noise signal $u[n]$ with variance $\sigma_1^2 = 0.01$ through a linear time-invariant filter $A(z)$ whose pole zero plot is depicted below (notice the double poles and zeros). Determine and plot the power spectral density function $S_{ss}(\omega)$ of $s[n]$.



$$A(z) = z^2 / (z-0.95)^2$$

b. Now consider the situation in which we observe a realization of a signal $s[n]$ in additive Gaussian white noise $w[n]$; that is, we measure

$$x[n] \; = \; s[n] \; + \; w[n].$$

Assume that $w[n]$ has variance $\sigma_2^2 = 0.5$ and that $w[n]$ is independent of $u[n]$ (and hence independent of the signal $s[n]$). Design an optimal Wiener filter for estimating $s[n]$ from the measured noisy signal $x[n]$. Give an expression for the Wiener filter $H(\omega)$ in the frequency domain.

c. Implement the signal generation and Wiener filtering process in Matlab.

i. Generate $s[n]$ by applying a time-domain difference equation to a Gaussian white noise sequence $u[n]$.

ii. Generate $x[n]$ by adding another, independent Gaussian white noise sequence $w[n]$ to $s[n]$ (take care to scale the noises to the proper power levels).

iii. Based on the Wiener filter $H(\omega)$ you derived above, devise an FFT-based procedure for filtering $x[n]$. Compare the results of your Wiener filtering to the true signal $s[n]$.

iv. Experiment with different power levels of the noise $w[n]$. How does σ_2^2 affect the output of the Wiener filter? What happens as $\sigma_2^2 \to 0$ and $\sigma_2^2 \to \infty$?