# ECE 431 Digital Signal Processing
# Midterm Exam I — Practice Problems

**0.** An LTI system has impulse response $h[n] = 5(-1/2)^n u[n]$. Use the DTFT to find the output of this system when the input is $x[n] = (1/3)^n u[n]$.

**1.** We obtain a DT signal $x[n]$ by sampling a CT signal $x(t)$. Unfortunately, we do not sample often enough and aliasing occurs. Is the DTFT of $x[n]$ periodic in frequency $\omega$? (Yes or No — and explain)

**2.** Suppose we want to compute the DFT of an $N$-point signal. Roughly speaking, by what factor does the FFT reduce the computational complexity if $N = 4$? If $N = 1024$?

**3.** Consider the CT signal $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$, where $f_2 = 3f_1$. Sample $x(t)$ at sampling frequency $f_s = 3f_1$ to obtain a DT signal $x[n]$.

a. What are the corresponding "digital" frequencies $\omega_1$ and $\omega_2$ (in radians/sample)? **Does aliasing occur?**

b. Suppose we are only interested in the cosine at frequency $\omega_1$. Sketch the frequency response of a DT filter that *perfectly* recovers $\cos(\omega_1 n)$ from $x[n]$ **and** show how the filter you propose can be constructed from lowpass filters alone.

**4.** Let $H(\omega) = \cos(\omega/2)$ be the frequency response of a DT LTI system with impulse response $h[n]$.

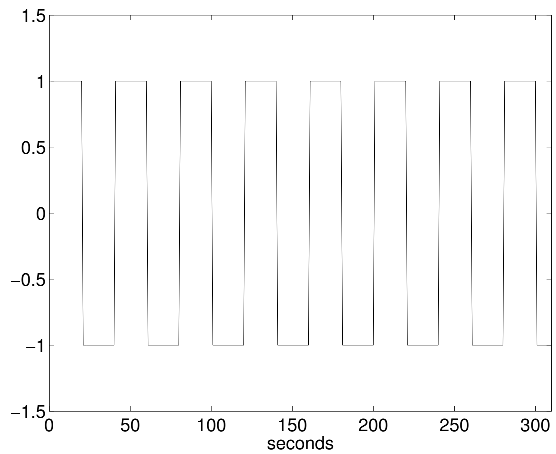
a. Sketch $H(\omega)$ on the interval $-\pi \leq \omega \leq \pi$. **Make sure to label axes**.

b. Let $x[n] = 0.5\sin(\pi n) + 2\cos(\frac{2}{3}\pi n)$ be the input to the system. Compute the output $y[n] = h[n] * x[n]$.

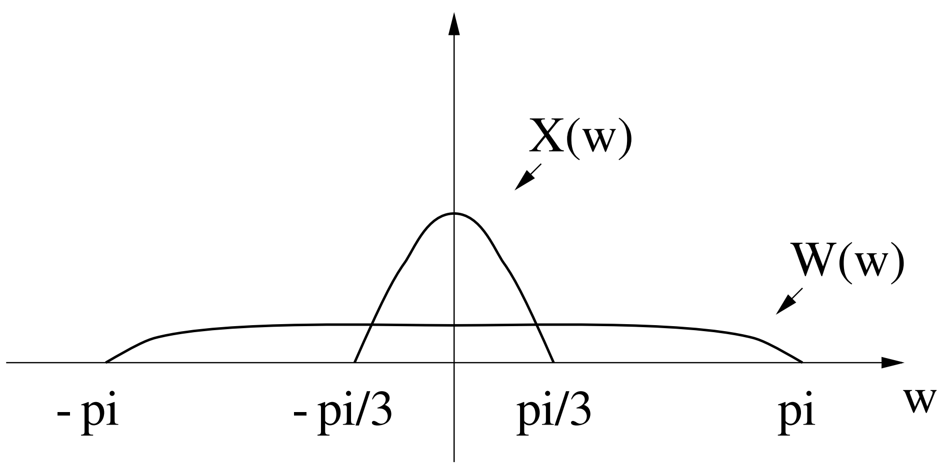
**5.** As a new employee at *Speechosonixs, Inc.*, our boss orders us to design an ideal digital filter system to bandpass filter speech signals and leave only the portions of the signal in the ranges from -2 kHz to -1 kHz and from 1 kHz to 2 kHz. Using a sampling rate of 8 kHz (and assuming ideal A/D and D/A) specify a block diagram for the system. Be sure to include your specifications (cut-off frequencies on $-\pi \leq \omega \leq \pi$) for the ideal discrete-time filter.

**6.** Let the sequence $X[k]$ be the DFT of a signal $x[n]$. If we conjugate $X[k]$ and take its DFT again to obtain a new sequence $Y[k]$, how is the sequence $Y[k]$ related to the original DT signal $x[n]$?

**7.** Consider the periodic CT square wave signal $x_c(t)$ depicted below. How fast must we sample $x_c(t)$ to avoid aliasing?




**8.** The DTFTs of a DT signal $X(\omega)$ and noise $W(\omega)$ are pictured below.




We observe the DT signal plus noise in our samples:

$$y[n] = x[n] + w[n]$$

You are asked to design a filter to reduce the noise in $y[n]$. Our boss is very frugal, and insists that you use the highpass filter

$$H(\omega) = 0,\ |\omega| < 2\pi/5, \quad H(\omega) = 1,\ 2\pi/5 \leq |\omega| < \pi$$

which was left over from a previous application. Can you design a good noise removal filter using $H(\omega)$?