Active Sensing and Learning

ICASSP 2011, May 23, Prague

www.ece.wisc.edu/~nowak/ASL.html

Jarvis Haupt
www.ece.umn.edu/~jdhaupt

Rob Nowak
www.ece.wisc.edu/~nowak
Adaptive Information

**Goal:** Estimate an unknown object $x \in \mathcal{X}$ from scalar samples

**Information:** samples of the form $y_1(x), \ldots, y_n(x)$, the values of certain functionals of $x$
Adaptive Information

**Goal:** Estimate an unknown object $x \in \mathcal{X}$ from scalar samples

**Information:** samples of the form $y_1(x), \ldots, y_n(x)$, the values of certain functionals of $x$

**Non-Adaptive Information:** $y_1, y_2, \cdots \in \mathcal{Y}$ non-adaptively chosen (deterministically or randomly) independent of $x$
Adaptive Information

**Goal:** Estimate an unknown object \( x \in \mathcal{X} \) from scalar samples

**Information:** samples of the form \( y_1(x), \ldots, y_n(x) \), the values of certain functionals of \( x \)

**Non-Adaptive Information:** \( y_1, y_2, \cdots \in \mathcal{Y} \) non-adaptively chosen (deterministically or randomly) independent of \( x \)

**Adaptive Information:** \( y_1, y_2, \cdots \in \mathcal{Y} \) are selected sequentially and \( y_i \) can depend on previously gathered information, i.e., \( y_1(x), \ldots, y_{i-1}(x) \)
Adaptive Information

**Goal:** Estimate an unknown object $x \in X$ from scalar samples

**Information:** samples of the form $y_1(x), \ldots, y_n(x)$, 
the values of certain functionals of $x$

**Non-Adaptive Information:** $y_1, y_2, \cdots \in \mathcal{Y}$ non-adaptively chosen (deterministically or randomly) independent of $x$

**Adaptive Information:** $y_1, y_2, \cdots \in \mathcal{Y}$ are selected sequentially and $y_i$ can depend on previously gathered information, i.e., $y_1(x), \ldots, y_{i-1}(x)$

Does adaptivity help?
Feedback from Data Analysis to Data Collection

\[ \mathcal{Y}: \text{possible measurements/experiments} \]

\[ \mathcal{X}: \text{models/hypotheses under consideration} \]

\[ y_1(x), y_2(x), \ldots: \text{information/data} \]
Feedback from Data Analysis to Data Collection

\[ \mathcal{Y}: \text{possible measurements/experiments} \]

\[ \mathcal{X}: \text{models/hypotheses under consideration} \]

\[ y_1(x), y_2(x), \ldots: \text{information/data} \]
Feedback from Data Analysis to Data Collection

\( \mathcal{X} \): models/hypotheses under consideration

\( \mathcal{Y} \): possible measurements/experiments

\( y_1(x), y_2(x), \ldots \): information/data
Feedback from Data Analysis to Data Collection

\[ \mathcal{X}: \text{models/hypotheses under consideration} \]

\[ \mathcal{Y}: \text{possible measurements/experiments} \]

\[ y_1(x), y_2(x), \ldots : \text{information/data} \]
Outline of Tutorial

Part 1: Introduction (Rob), 9:00-9:30

Part 2: Active Sensing (Jarvis), 9:30-10:30

Break, 10:30-10:45

Part 3: Active Learning (Rob), 10:45-11:45

Part 4: Conclusions and Future Directions, 11:45-12

Outline of Part 1:

Sequential Experimental Design
Adaptive Sensing for Sparse Recovery
Sensing and Inference in Large Networked Systems
Active Learning in Machines and Humans
Mathematics of Active Sensing and Learning

Friday, May 20, 2011
Decided to make new astronomical measurements when "the discrepancy between prediction and observation [was] large enough to give a high probability that there is something new to be found." Jaynes (1986)
The Scientific Process in a Laboratory

experiments

scientist

data
The Scientific Process at Large
The Scientific Process at Large
The Scientific Process at Large
The Scientific Process at Large
Motivation: Inferring Biological Pathways
Motivation: Inferring Biological Pathways

virus

fruit fly

Paul Alhquist  
(Molecular Virology)

Audrey Gasch  
(Genetics)
Motivation: Inferring Biological Pathways

virus

13,071 single-gene knock-down cell strains

fruit fly
Motivation: Inferring Biological Pathways

13,071 single-gene knock-down cell strains

infect each strain with fluorescing virus

microwell array
Motivation: Inferring Biological Pathways

13,071 single-gene knock-down cell strains

Infecting virion RNA

1. Regulated viral translation
   LSM1-LSM7, PAT1, DHH1, DED1, RPL19b, RPA1, RPA34, RRN3

2. Protein targeting, regulated stability
   SCS2, PRE9

3. RNA template recruitment from translation to replication
   LSM1-LSM7, PAT1, DHH1, SCP160

Viral RNA

4. Membrane synthesis, trafficking, lipid composition
   OLE1, ACB1, DRS2, RCD1, NEM1, SPO7

5. Chaperone activation of replication complex
   YDJ1 (HSP70,90)

6. Survival, fate of progeny (+)RNA
   SKI2,3,7,8

Infect each strain with fluorescing virus

microwell array

fruit fly

Paul Alhquist
(Molecular Virology)

Audrey Gasch
(Genetics)
Motivation: Inferring Biological Pathways

“Drosophila RNAi screen identifies host genes important for influenza virus replication,” Nature 2008. How do they confidently determine the ~100 out of 13K genes hijacked for virus replication from extremely noisy data?
Motivation: Inferring Biological Pathways

“Drosophila RNAi screen identifies host genes important for influenza virus replication,” Nature 2008. How do they confidently determine the ~100 out of 13K genes hijacked for virus replication from extremely noisy data?

Sequential Experimental Design:

**Stage 1**: assay all 13K strains, twice; keep all with significant fluorescence in one or both assays for 2nd stage (13K → 1K)

**Stage 2**: assay remaining 1K strains, 6-12 times; retain only those with statistically significant fluorescence (1K → 100)
Motivation: Inferring Biological Pathways

“Drosophila RNAi screen identifies host genes important for influenza virus replication,” Nature 2008. How do they confidently determine the ~100 out of 13K genes hijacked for virus replication from extremely noisy data?

Sequential Experimental Design:

**Stage 1:** assay all 13K strains, twice; keep all with significant fluorescence in one or both assays for 2nd stage (13K → 1K)

**Stage 2:** assay remaining 1K strains, 6-12 times; retain only those with statistically significant fluorescence (1K → 100)

vastly more efficient than replicating all 13K experiments many times
Feedback from Data Analysis to Data Collection

- high-throughput experiments
- experiment space
- data
- model space
- sets of genes critical to a certain function/process
- microarray or assay datasets
Feedback from Data Analysis to Data Collection

- high-throughput experiments
- model space
  - sets of genes critical to a certain function/process
- experiment space
- data
  - microarray or assay datasets
Feedback from Data Analysis to Data Collection

sets of genes critical to a certain function/process

model space

experiment space

high-throughput experiments

data

microarray or assay datasets
Feedback from Data Analysis to Data Collection

sets of genes critical to a certain function/process

high-throughput experiments

experiment space

model space

data
microarray or assay datasets
Adaptive Sensing for Sparse Recovery
(image reconstruction, compressed sensing, inverse problems)

\[ y = Ax + w, \text{ with } A \in \mathbb{R}^{m \times n}, x \in \mathbb{R}^n \text{ (but sparse)}, w \sim \mathcal{N}(0, I) \]

\textbf{Goal:} recover \( x \) from \( y \)
Adaptive Sensing for Sparse Recovery
(image reconstruction, compressed sensing, inverse problems)

\[ y = A x + w, \text{ with } A \in \mathbb{R}^{m \times n}, \ x \in \mathbb{R}^n \ (\text{but sparse}), \ w \sim \mathcal{N}(0, I) \]

Goal: recover \( x \) from \( y \)

Is sequentially designing (rows of) \( A \) advantageous?
Motivation: Randomized Experiments

\[ \tilde{y} = \underbrace{Y}_{k \times 1} \times \begin{array}{c} \text{sparse} \\ \text{signal} \end{array} + \begin{array}{c} \text{noise} \end{array} \]

indirect (randomized) measurement
Motivation: Randomized Experiments

\[ \tilde{y} = \begin{bmatrix} k \times 1 \end{bmatrix} \times \text{sparse signal} + \text{noise} \]

indirect (randomized) measurement

put mixtures of single-deletion strains into each well
$y = \phi x$ where $\phi \in \mathbb{R}^{m \times n}$, $x \in \mathbb{R}^n$ (but sparse), $w \sim \mathcal{N}(0, I)$

Experimental design question: choice of $A_1, \ldots, A_k$ to maximize probability of correctly identifying $x$.

A visual representation of the math behind compressive sensing.

Look cool without breaking the bank. Our durable, high-quality, pre-shrunk 100% cotton t-shirt is what to wear when you want to go comfortably casual. Preshrunk, durable and guaranteed.

- 5.6 oz. 100% cotton
- Standard fit
Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)
Sensing and Inference in Large Networked Systems

Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)

Social Networks
Sensing and Inference in Large Networked Systems

Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)

Social Networks

Biological Networks
(JMDBase)
Sensing and Inference in Large Networked Systems

Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)

Social Networks

Biological Networks
(JMDBase)

Brain Networks
(Worsley et al, 2005)
Sensing and Inference in Large Networked Systems

Technological Networks
(Internet Mapping Project, US power grid, UCLA CENS)

Social Networks

Biological Networks
(JMDBase)

Brain Networks
(Worsley et al, 2005)

Challenges:
• Inferring structure & function of the system
• Optimized design & resource allocation
• Pattern analysis & anomaly detection
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.

Network(s) of interactions can be revealed via clustering based on measured features.
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.

Network(s) of interactions can be revealed via clustering based on measured features:

- genes and expression/interaction profiles
- network routers and traffic/distance profiles
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.

Network(s) of interactions can be revealed via clustering based on measured features

**Similarity-Based Clustering:** Each component (gene/router) has an associated feature (measurement profile). Components can be clustered based on feature similarities.
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.

Network(s) of interactions can be revealed via clustering based on measured features

**genes and expression/interaction profiles**

**network routers and traffic/distance profiles**

**Similarity-Based Clustering:** Each component (gene/router) has an associated feature (measurement profile). Components can be clustered based on feature similarities.

**Recent Result:** A sequential method for selecting “informative” similarities that produces accurate clusters from as few as $3N \log N$ similarities.
Cognitive Radio Spectrum Sensing

“primary” users have preference over “secondary” users
Cognitive Radio Spectrum Sensing

“primary” users have preference over “secondary” users

most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users “sense” spectrum to find an unoccupied channel
Cognitive Radio Spectrum Sensing

“primary” users have preference over “secondary” users

most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users “sense” spectrum to find an unoccupied channel

**Goal**: Find open channel(s) as quickly as possible. Two approaches:

1) listen to each channel for a fixed amount of time and make decision
2) listen to each channel for a *data-adaptive* amount of time to make decisions as quickly as possible
Cognitive Radio Spectrum Sensing

“primary” users have preference over “secondary” users

most channels occupied by primary users, but they come and go in unpredictable manner. Secondary users “sense” spectrum to find an unoccupied channel.

**Goal**: Find open channel(s) as quickly as possible. Two approaches:

1) listen to each channel for a fixed amount of time and make decision
2) listen to each channel for a data-adaptive amount of time to make decisions as quickly as possible

adaptive spectrum sensing is significantly more time-efficient than fixed sensing.

Friday, May 20, 2011
Active Learning in Machines and Humans

model space

data collection

data

Friday, May 20, 2011
Active Learning in Machines and Humans

Sensing

Computing

Model space

Data collection

Data
Active Learning in Machines and Humans

model space

data collection

data

Sensing
Computing
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$. 

![Graph showing cholesterol and BMI with heart disease status]
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$. 

![Graph showing heart disease prediction based on BMI and cholesterol levels with a best linear classifier.](image)
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$

Passive Learning: training examples selected at random
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$

Passive Learning: training examples selected at random

Active Learning: especially informative examples are sequentially selected
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$

Passive Learning: training examples selected at random

Active Learning: especially informative examples are sequentially selected
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^{n}$

Passive Learning: training examples selected at random

Active Learning: especially informative examples are sequentially selected
Active Learning

Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^n$

Active Learning: especially informative examples are sequentially selected

Passive Learning: training examples selected at random

Active Learning: especially informative examples are sequentially selected
Learn to predict labels $y$ from features $x$ based on training examples $\{(x_i, y_i)\}_{i=1}^{n}$.

Active Learning: especially informative examples are sequentially selected.

Passive Learning: training examples selected at random.

Active Learning can very effectively “narrow down” the location of the optimal decision boundary.
The Theory of the Organism-Environment System: III. Role of Efferent Influences on Receptors in the Formation of Knowledge*

Timo Jarvilehto
Department of Behavioral Sciences, University of Oulu, Finland

Abstract—The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, which makes the development of a dynamic living system, the organism-environment system, possible. In this connection process, the efferent influences on receptor activity are of particular significance because, with their help, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of the efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.
Abstract—The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, which makes the development of a dynamic living system, the organism-environment system, possible. In this connection process, the efferent influences on receptor activity are of particular significance because, with their help, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of the efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.
Abstract—The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, which makes the development of a dynamic living system, the organism-environment system, possible. In this connection process, the efferent influences on receptor activity are of particular significance because, with their help, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of the efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.
Abstract—The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, that makes the develop of process, the efferent influences on receptor activity are of particular significance because, with their help, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of the efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.
Sensing

Computing

The present article is an attempt to give—in the frame of the theory of the organism-environment system (Jarvilehto, 1998a)—a new interpretation to the role of efferent influences on receptor activity and to the functions of senses in the formation of knowledge. It is argued, on the basis of experimental evidence and theoretical considerations, that the senses are not transmitters of environmental information, but create a direct connection between the organism and the environment, that makes the development of process, the efferent influences on receptor activity are of particular significance because, the receptors may be adjusted in relation to the parts of the environment that are most important in achieving behavioral results. Perception is the process of joining of new parts of the environment to the organism-environment system; thus, the formation of knowledge by perception is based on reorganization (widening and differentiation) of the organism-environment system, and not on transmission of information from the environment. With the help of efferent influences on receptors, each organism creates its own peculiar world that is simultaneously subjective and objective. The present considerations have far-reaching influences as well on experimental work in neurophysiology and psychology of perception as on philosophical considerations of knowledge formation.
Visual Perception

Attentional mechanisms probably limit our capacity to about 44 bits per-glimpse (Verghese and Pelli (1992))

So how do we perceive ‘reality’ from so few bits of information?
Visual Perception

Attentional mechanisms probably limit our capacity to about 44 bits per-glimpse (Verghese and Pelli (1992))

So how do we perceive ‘reality’ from so few bits of information?

Churchland, Ramachandran, & Sejnowski ’94: “Interactive vision is exploratory and predictive. Visual learning allows an animal to predict what will happen in the future; behavior, such as eye movements, aids in updating and upgrading the predictive representations.”
Attentional mechanisms probably limit our capacity to about 44 bits per-glimpse (Verghese and Pelli (1992))

So how to we perceive ‘reality’ from so few bits of information?

Churchland, Ramachandran, & Sejnowksi ’94: “Interactive vision is exploratory and predictive. Visual learning allows an animal to predict what will happen in the future; behavior, such as eye movements, aids in updating and upgrading the predictive representations.”
Mathematical Theory of Active Sensing and Learning

\[ \mathcal{Y}: \text{possible measurements/experiments} \]

\[ \mathcal{X}: \text{models/hypotheses under consideration} \]

\[ y_1(x), y_2(x), \ldots: \text{information/data} \]
Adaptive vs. Non-Adaptive: Three Situations

The “bare minimum” number of measurements depends on intrinsic complexity of $\mathcal{X}$. In practice, the minimum number depends on jointly on $\mathcal{X}$ and $\mathcal{Y}$.
Adaptive vs. Non-Adaptive: Three Situations

The “bare minimum” number of measurements depends on intrinsic complexity of $\mathcal{X}$. In practice, the minimum number depends on jointly on $\mathcal{X}$ and $\mathcal{Y}$.

Equal and Good:
adaptive and non-adaptive equally informative and require about the bare minimum of measurements
Adaptive vs. Non-Adaptive: Three Situations

The “bare minimum” number of measurements depends on intrinsic complexity of $\mathcal{X}$. In practice, the minimum number depends on jointly on $\mathcal{X}$ and $\mathcal{Y}$. 

**Equal and Good:**

Adaptive and non-adaptive equally informative and require about the bare minimum of measurements.

**Equal and Bad:**

Adaptive and non-adaptive equally (non)-informative and require many more measurements than the bare minimum.
Adaptive vs. Non-Adaptive: Three Situations

The “bare minimum” number of measurements depends on intrinsic complexity of $X$. In practice, the minimum number depends on jointly on $X$ and $Y$.

**Equal and Good:**
adaptive and non-adaptive equally informative and require about the bare minimum of measurements

**Equal and Bad:**
adaptive and non-adaptive equally (non)-informative and require many more measurements than the bare minimum

**Good and Bad:**
adaptive requires bare minimum number of measurements, non-adaptive requires many more
Assume $\mathcal{X}$ is equipped with metric $d$ and is compact.
Assume $\mathcal{X}$ is equipped with metric $d$ and is compact.

Let $\mathcal{X}_\varepsilon \subset \mathcal{X}$ be a finite subset of size $N_\varepsilon$ having the property that any element of $\mathcal{X}$ is within distance $\varepsilon$ of an element in $\mathcal{X}_\varepsilon$. 
Assume $\mathcal{X}$ is equipped with metric $d$ and is compact.

Let $\mathcal{X}_\epsilon \subset \mathcal{X}$ be a finite subset of size $N_\epsilon$ having the property that any element of $\mathcal{X}$ is within distance $\epsilon$ of an element in $\mathcal{X}_\epsilon$

**Metric Entropy:** Need at least $\log N_\epsilon$ bits of information to approximately determine any $x \in \mathcal{X}$
Assume $\mathcal{X}$ is equipped with metric $d$ and is compact.

Let $\mathcal{X}_\epsilon \subset \mathcal{X}$ be a finite subset of size $N_\epsilon$ having the property that any element of $\mathcal{X}$ is within distance $\epsilon$ of an element in $\mathcal{X}_\epsilon$.

**Metric Entropy:** Need at least $\log N_\epsilon$ bits of information to approximately determine any $x \in \mathcal{X}$.

Ex. suppose $\mathcal{X} = [0, 1]^d$. we can take a uniform grid of points spaced $\epsilon$ apart as our cover. Then $N_\epsilon = \left(\frac{1}{\epsilon}\right)^d$ and $\log N_\epsilon = d \log(1/\epsilon)$. 
Binary Search

\[ x = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]

\[ y = \text{“membership queries”} \]
Binary Search

\[ \mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]

\[ \mathcal{Y} = \text{“membership queries”} \]
Binary Search

\[ x = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]

\[ y = \text{“membership queries”} \]

**binary search**: sequentially select queries
Binary Search

\( \mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \)

\( \mathcal{Y} = \) “membership queries”

**binary search**: sequentially select queries

\[ 1/3 = 0... \]
Binary Search

\[ \mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]
\[ \mathcal{Y} = \text{“membership queries”} \]

**binary search**: sequentially select queries

\[ \frac{1}{3} = 01\ldots \]
Binary Search

\[ X = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]

\[ Y = \text{“membership queries”} \]

\text{binary search}: \text{sequentially select queries}

\[ 1/3 = 010\ldots \]
$\mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \}$

$\mathcal{Y} = \text{“membership queries”}$

**binary search**: sequentially select queries

$\frac{1}{3} = 0101\ldots$
\[ \mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \} \]

\[ \mathcal{Y} = \text{“membership queries”} \]

**Binary search:** sequentially select queries

\[ \frac{1}{3} = 0101\ldots \]

requires \( \log_2 N \) queries
Binair Search

\[ \mathcal{X} = \left\{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \right\} \]

\[ \mathcal{Y} = \text{“membership queries”} \]

**Binary search:** sequentially select queries

\[ \frac{1}{3} = 0101\ldots \]

requires \( \log_2 N \) queries

**Linear search:** query points uniformly (possibly random)
$\mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \}$

$\mathcal{Y} = \text{“membership queries”}$

**Binary Search**

**binary search**: sequentially select queries

$\frac{1}{3} = 0101\ldots$  \hspace{1cm} requires $\log_2 N$ queries

**linear search**: query points uniformly (possibly random)
$\mathcal{X} = \{ \text{subsets } [0, \frac{1}{N}], [0, \frac{2}{N}], \ldots, [0, 1] \}$

$\mathcal{Y} = \text{“membership queries”}$

**binary search**: sequentially select queries

$\frac{1}{3} = 0101\ldots$ requires $\log_2 N$ queries

**linear search**: query points uniformly (possibly random)

requires $O(N)$ queries
Outline of Tutorial

Part 1: Introduction (Rob), 9:00-9:30

Part 2: Active Sensing (Jarvis), 9:30-10:30

Break, 10:30-10:45

Part 3: Active Learning (Rob), 10:45-11:45

Part 4: Conclusions and Future Directions, 11:45-12